The sine and cosine rules (answers at the end)

1 The shortest side of a triangle is 4.3 m long. Two of the angles are 45.1° and 51.2°. Find the length of the longest side.

2 In triangle ABC the length $BC = 15.1$ cm, angle $BAC = 56^\circ$, angle $ABC = 73^\circ$. Calculate the lengths of the sides AB and AC.

3 The length of the longest side of a triangle is 15 cm. Two of the angles are 39° and 48°. Find the length of the shortest side.

4 In a triangle XYZ find angle Z when $YZ = 4.7$ cm, $XZ = 10.5$ cm and $XY = 8.9$ cm.

5 The sides of a triangle are 7 cm, 9 cm and 12 cm. Find its angles and its area.

6 In triangle LMN, $LN = 8.6$ cm, $LM = 9.9$ cm, $L = 75^\circ$, find the length MN, and the angles M and N.

7 In a quadrilateral $ABCD$, $AB = 4$ cm, $BC = 5$ cm, $CD = 7$ cm, $DA = 5$ cm and angle ABC is 87°. Find angle ADC.

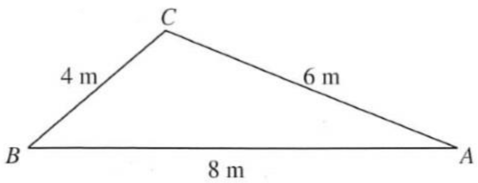
8 In a quadrilateral $PQRS$, $PQ = 4$ cm, $QR = 3$ cm and $RS = 4$ cm. Angle $PQR = 92^\circ$ and angle $QRS = 110^\circ$. Find the lengths of the diagonals and the length PS.

9 In a quadrilateral $ABCD$, $AB = 3$ cm, $BC = 4$ cm, $CD = 7$ cm, $DA = 8$ cm and the diagonal $AC = 6$ cm. Find the area of the quadrilateral.

10 A small weight W is supported by two strings of lengths 1 metre and 1.2 metres from two points 1.4 metres apart on a horizontal ceiling. How far below the ceiling is W?

11 Two ships leave a harbour at the same time. The first steams on a bearing 045° at 16 km h^{-1} and the second on a bearing 305° at 18 km h^{-1}. How far apart will they be after 2 hours?

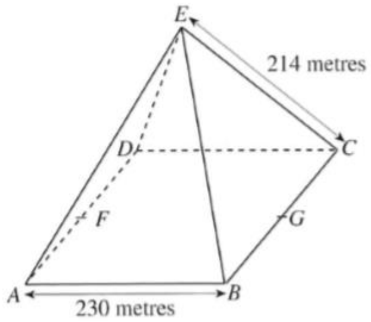
12 The diagram, which is not drawn to scale, shows a triangular flower bed ABC of which the sides BC, CA and AB are 4 metres, 6 metres and 8 metres long respectively.

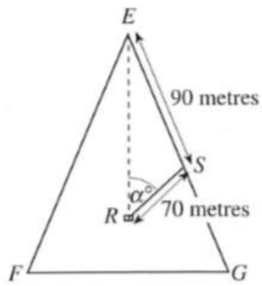


(a) Calculate the size, in degrees, of the angle at A.

(b) Calculate the area of the flower bed ABC. (OCR)

13 The first diagram shows a sketch of the Great Pyramid at Giza in Egypt. It can be modelled as a square-based pyramid, where $ABCD$ is a horizontal square of side 230 metres. The faces ABE, BCE, CDE and ADE can be considered as isosceles triangles, in which the sides of equal length are 214 metres.





(a) Calculate the vertical height of the pyramid; that is, the height of E above the square base $ABCD$.

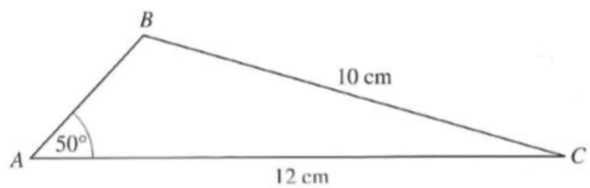
The second diagram shows a sketch of a vertical cross section through the pyramid. The vertical plane of the cross section passes through the mid-points of the sides AD and BC of the base of the pyramid. These mid-points are labelled F and G in both diagrams. A burial chamber is at R vertically below the apex of the pyramid. A ventilation shaft slopes upwards from the burial chamber to meet the face BCE of the pyramid, as shown on the second diagram. This shaft meets the sloping face of the pyramid at a point S on the line EG.

The ventilation shaft, RS is 70 metres long and it meets the face of the pyramid at S so that ES is 90 metres.

(b) Calculate the angle, α, that the ventilation shaft makes with the vertical.

(c) Calculate the depth of the burial chamber, R, below the vertex, E, of the pyramid. (OCR)

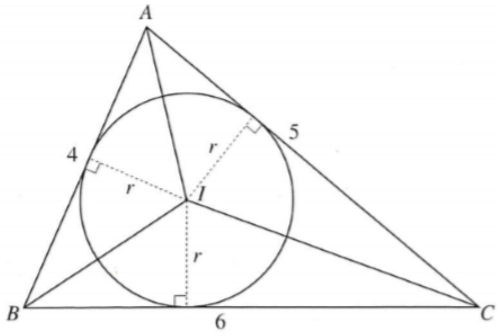
14 The diagram, which is not drawn to scale, shows an obtuse-angled triangle ABC in which $AC = 12$ cm, angle $A = 50°$ and $BC = 10$ cm.



(a) Calculate the size of angle C.

(b) Calculate the area of triangle ABC. (OCR, adapted)

15 (a) A triangle ABC has sides BC, CA and AB of lengths 6 cm, 5 cm and 4 cm respectively. Calculate the area of the triangle.



(b) The diagram shows the triangle ABC together with a circle, centre I, drawn inside the triangle ABC and touching the sides BC, CA and AB.

By considering the sum of the areas of triangles IBC, ICA and IAB, or otherwise, calculate the radius, r cm, of the circle. (OCR, adapted)

16* Two points P and Q are visible from point A but are inaccessible from A. The bearings of P and Q from A are 015° and 034° respectively, and their bearings from B, which is 100 m east of A, are 305° and 325°. Calculate the length PQ.

17* In a triangle ABC, suppose that $A > B$ and consider two cases.

(a) If both angles are acute, use the definition of $\sin\theta°$, or the graph of $\sin\theta°$, to explain why $\sin A > \sin B$.

(b) If the angle at A is obtuse, explain why $(180° - A) > B$, and deduce again that $\sin A > \sin B$.

Hence use the sine rule to show that, in either case, $a > b$.

18* In a triangle ABC, suppose that $a > b$. Use the sine rule to show that $\sin A > \sin B$, and consider three cases.

(a) If the angles at A and B are both acute, explain why $A > B$.

(b) If the angle at A is obtuse and the angle at B is acute, explain why $A > B$.

(c) If the angle at A is acute and the angle at B is obtuse, show that $A > (180° - B)$, and explain why this is impossible.

Use your results to show that, if $a > b$, then $A > B$.

1 6.03 m

2 14.2 cm, 17.4 cm

3 9.45 cm

4 57.4°

5 35.4°, 48.2°, 96.4°, 31.3 cm^2

6 11.3 cm, 47.3°, 57.7°

7 59.9°

8 5.08 cm, 5.76 cm, 4.51 cm

9 25.7 cm^2

10 0.840 m

11 52.2 km

12 (a) 29.0° (b) 11.6 m^2

13 (a) 139 m (b) 55.0° (c) 110 m

14 (a) 16.8° (b) 17.4 cm^2

15 (a) 9.92 cm^2 (b) 1.32 cm

16 36.0 m