The Binomial theorem (answers at the end)

1 Expand $(3+4x)^3$.

2 Find the first three terms in the expansions, in ascending powers of x, of

(a) $(1+4x)^{10}$, (b) $(1-2x)^{16}$.

3 Find the coefficient of a^3b^5 in the expansions of

(a) $(3a-2b)^8$, (b) $\left(5a+\frac{1}{2}b\right)^8$.

4 Expand $(3+5x)^7$ in ascending powers of x up to and including the term in x^2. By putting $x=0.01$, find an approximation, correct to the nearest whole number, to 3.05^7.

5 Obtain the first four terms in the expansion of $\left(2+\frac{1}{4}x\right)^8$ in ascending powers of x. By substituting an appropriate value of x into this expansion, find the value of 2.0025^8 correct to three decimal places. (OCR)

6 Find, in ascending powers of x, the first three terms in the expansion of $(2-3x)^8$. Use the expansion to find the value of 1.997^8 to the nearest whole number. (OCR)

7 Expand $\left(x^2+\dfrac{1}{x}\right)^3$, simplifying each of the terms.

8 Expand $\left(2x-\dfrac{3}{x^2}\right)^4$.

9 Expand and simplify $\left(x+\dfrac{1}{2x}\right)^6+\left(x-\dfrac{1}{2x}\right)^6$. (OCR)

10 Find the coefficient of x^2 in the expansion of $\left(x^4+\dfrac{4}{x}\right)^3$.

11 Find the term independent of x in the expansion of $\left(2x+\dfrac{5}{x}\right)^6$.

12 Find the coefficient of y^4 in the expansion of $(1+y)^{12}$. Deduce the coefficient of

(a) y^4 in the expansion of $(1+3y)^{12}$,

(b) y^8 in the expansion of $(1-2y^2)^{12}$,

(c) x^8y^4 in the expansion of $(x+\frac{1}{2}y)^{12}$.

13 Determine the coefficient of p^4q^7 in the expansion of $(2p-q)(p+q)^{10}$.

14 Find the first three terms in the expansion of $(1+2x)^{20}$. By substitution of a suitable value of x in each case, find approximations to

(a) 1.002^{20}, (b) 0.996^{20}.

15 Write down the first three terms in the binomial expansion of $\left(2-\dfrac{1}{2x^2}\right)^{10}$ in ascending powers of $\dfrac{1}{x}$. Hence find the value of 1.995^{10} correct to three significant figures. (OCR)

16 Two of the following expansions are correct and two are incorrect. Find the two expansions which are incorrect.

A: $(3+4x)^5=243+1620x+4320x^2+5760x^3+3840x^4+1024x^5$

B: $(1-2x+3x^2)^3=1+6x-3x^2+28x^3-9x^4+54x^5-27x^6$

C: $(1-x)(1+4x)^4=1+15x+80x^2+160x^3-256x^5$

D: $(2x+y)^2(3x+y)^3=108x^5+216x^4y+171x^3y^2+67x^2y^3+13xy^4+y^6$

17 Find and simplify the term independent of x in the expansion of $\left(\dfrac{1}{2x}+x^3\right)^8$. (OCR)

18 Find the term independent of x in the expansion of $\left(2x+\dfrac{1}{x^2}\right)^9$.

19 Evaluate the term which is independent of x in the expansion of $\left(x^2-\dfrac{1}{2x^2}\right)^{16}$. (OCR)

20 Find the coefficient of x^{-12} in the expansion of $\left(x^3-\dfrac{1}{x}\right)^{24}$. (OCR)

21 Expand $(1+3x+4x^2)^4$ in ascending powers of x as far as the term in x^2. By substituting a suitable value of x, find an approximation to 1.0304^4.

22 Expand and simplify $(3x+5)^3-(3x-5)^3$.
Hence solve the equation $(3x+5)^3-(3x-5)^3=730$.

23 Solve the equation $(7-6x)^3+(7+6x)^3=1736$.

24 (a) Show that

(i) $\binom{6}{4}=\binom{6}{2}$, (ii) $\binom{10}{3}=\binom{10}{7}$, (iii) $\binom{15}{12}=\binom{15}{3}$, (iv) $\binom{13}{6}=\binom{13}{7}$.

(b) State the possible values of x in each of the following.

(i) $\binom{11}{4}=\binom{11}{x}$ (ii) $\binom{16}{3}=\binom{16}{x}$ (iii) $\binom{20}{7}=\binom{20}{x}$ (iv) $\binom{45}{17}=\binom{45}{x}$

(c) Use the definition $\binom{n}{r}=\dfrac{n!}{r!(n-r)!}$ to prove that $\binom{n}{r}=\binom{n}{n-r}$.

25 Find the value of 1.0003^{18} correct to 15 decimal places.

26 (a) Expand $(2\sqrt{2}+\sqrt{3})^4$. Give your answer in the form $a+b\sqrt{6}$, where a and b are integers.

(b) Find the exact value of $(2\sqrt{2}+\sqrt{3})^5$.

27 (a) Expand and simplify $(\sqrt{7}+\sqrt{5})^4+(\sqrt{7}-\sqrt{5})^4$. By using the fact that $0<\sqrt{7}-\sqrt{5}<1$, state the consecutive integers between which $(\sqrt{7}+\sqrt{5})^4$ lies.

(b) Without using a calculator, find the consecutive integers between which the value of $(\sqrt{3}+\sqrt{2})^6$ lies.

28* Find, in ascending powers of t, the first three terms in the expansions of

(a) $(1+\alpha t)^5$, (b) $(1-\beta t)^8$.

Hence find, in terms of α and β, the coefficient of t^2 in the expansion of $(1+\alpha t)^5(1-\beta t)^8$. (OCR)

29* (a) Show that

(i) $4\times\binom{6}{2}=3\times\binom{6}{3}=6\times\binom{5}{2}$, (ii) $3\times\binom{7}{4}=5\times\binom{7}{5}=7\times\binom{6}{4}$.

(b) State numbers a, b and c such that

(i) $a\times\binom{8}{5}=b\times\binom{8}{6}=c\times\binom{7}{5}$, (ii) $a\times\binom{9}{3}=b\times\binom{9}{4}=c\times\binom{8}{3}$.

(c) Prove that $(n-r)\times\binom{n}{r}=(r+1)\times\binom{n}{r+1}=n\times\binom{n-1}{r}$.

30* Prove that $\binom{n}{r-1}+2\binom{n}{r}+\binom{n}{r+1}=\binom{n+2}{r+1}$.

31* Find an expression, in terms of n, for the coefficient of x in the expansion

$$(1+4x)+(1+4x)^2+(1+4x)^3+\cdots+(1+4x)^n.$$

32* Given that

$$a+b(1+x)^3+c(1+2x)^3+d(1+3x)^3=x^3$$

for all values of x, find the values of the constants a, b, c and d.

1 $27 + 108x + 144x^2 + 64x^3$

2 (a) $1 + 40x + 720x^2$ (b) $1 - 32x + 480x^2$

3 (a) $-48\,384$ (b) $\frac{875}{4}$

4 $2187 + 25\,515x + 127\,575x^2$; 2455

5 $256 + 256x + 112x^2 + 28x^3$; 258.571

6 $256 - 3072x + 16\,128x^2$; 253

7 $x^6 + 3x^3 + 3 + \dfrac{1}{x^3}$

8 $16x^4 - 96x + \dfrac{216}{x^2} - \dfrac{216}{x^5} + \dfrac{81}{x^8}$

9 $2x^6 + 15\dfrac{x^2}{2} + \dfrac{15}{8x^2} + \dfrac{1}{32x^6}$

10 48

11 20 000

12 495; (a) 40 095 (b) 7920 (c) $\frac{495}{16}$

13 30

14 $1 + 40x + 760x^2$;

(a) 1.0408 (b) 0.9230

15 $1024 - \dfrac{2560}{x^2} + \dfrac{2880}{x^4}$; 999

16 B, D

17 $\frac{7}{16}$

18 5376

19 $\frac{6435}{128}$

20 -2024

21 $1 + 12x + 70x^2$; 1.127

22 $270x^2 + 250$; $\pm \frac{4}{3}$

23 $\pm \frac{5}{6}$

24 (b) (i) 4 or 7 (ii) 3 or 13

(iii) 7 or 13 (iv) 17 or 28

25 1.005 413 792 056 805

26 (a) $217 + 88\sqrt{6}$ (b) $698\sqrt{2} + 569\sqrt{3}$

27 (a) 568; 567 and 568 (b) 969 and 970

28 (a) $1 + 5\alpha t + 10\alpha^2 t^2$ (b) $1 - 8\beta t + 28\beta^2 t^2$

$10\alpha^2 - 40\alpha\beta + 28\beta^2$

29 (b) (i) $a = 3, b = 6, c = 8$

(ii) $a = 6, b = 4, c = 9$

31 $2n(n + 1)$

32 $a = -\frac{1}{6}, b = \frac{1}{2}, c = -\frac{1}{2}, d = \frac{1}{6}$