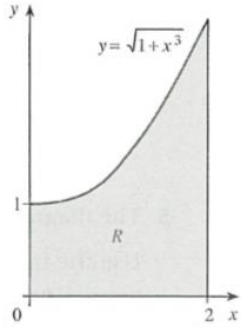
The trapezium rule (answers at the end)

1 Use the trapezium rule, with ordinates at $x = 1$, $x = 2$ and $x = 3$, to estimate the value of $\int_1^3 \sqrt{40 - x^3}\,dx$. (OCR)

2 The diagram shows the region R bounded by the curve $y = \sqrt{1 + x^3}$, the axes and the line $x = 2$. Use the trapezium rule with 4 intervals to obtain an approximation for the area of R, showing your working and giving your answer to a suitable degree of accuracy.

Explain, with the aid of a sketch, whether the approximation is an overestimate or an underestimate. (OCR)



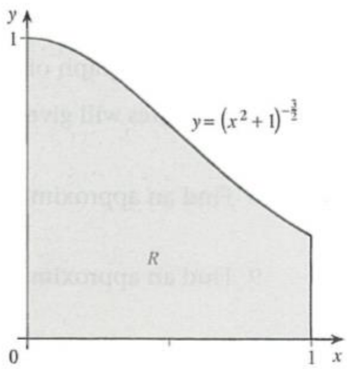
3 Use the trapezium rule with subdivisions at $x = 3$ and $x = 5$ to obtain an approximation to $\int_1^7 \frac{x^3}{1 + x^4}\,dx$, giving your answer correct to 3 places of decimals. (OCR)

4 Use the trapezium rule with 5 intervals to estimate the value of $\int_0^{0.5} \sqrt{1 + x^2}\,dx$, showing your working. Give your answer correct to 2 decimal places. (OCR)

5 The diagram shows the region R bounded by the axes, the curve $y = (x^2 + 1)^{-\frac{3}{2}}$ and the line $x = 1$. Use the trapezium rule, with ordinates at $x = 0$, $x = \frac{1}{2}$ and $x = 1$, to estimate the value of

$$\int_0^1 (x^2 + 1)^{-\frac{3}{2}}\,dx$$

giving your answer correct to 2 significant figures. (OCR)



6 A certain function f is continuous and is such that

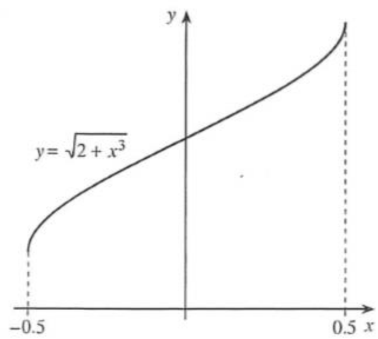
$f(2.0) = 15$, $f(2.5) = 22$, $f(3.0) = 31$,
$f(3.5) = 28$, $f(4.0) = 27$.

Use the trapezium rule to find an approximation to $\int_2^4 f(x)\,dx$.

7 The diagram shows a sketch of $y = \sqrt{2 + x^3}$ for values of x between -0.5 and 0.5.

(a) Use the trapezium rule, with ordinates at $x = -0.5$, $x = 0$ and $x = 0.5$ to find an approximate value for $\int_{-0.5}^{0.5} \sqrt{2 + x^3}\,dx$.

(b) Explain briefly, with reference to the diagram, why the trapezium rule can be expected to give a good approximation to the value of the integral in this case. (OCR)



8 The speeds of an athlete on a training run were recorded at 30-second intervals:

Time after start (s)	0	30	60	90	120	150	180	210	240
Speed ($m\,s^{-1}$)	3.0	4.6	4.8	5.1	5.4	5.2	4.9	4.6	3.8

The area under a speed–time graph represents the distance travelled. Use the trapezium rule to estimate the distance covered by the athlete, correct to the nearest 10 metres.

9 At a time t minutes after the start of a journey, the speed of a car travelling along a main road is v $km\,h^{-1}$. The table gives values of v every minute on the 10-minute journey.

t	0	1	2	3	4	5	6	7	8	9	10
v	0	31	46	42	54	57	73	70	68	48	0

Use the trapezium rule to estimate of the length of the 10-minute journey in kilometres.

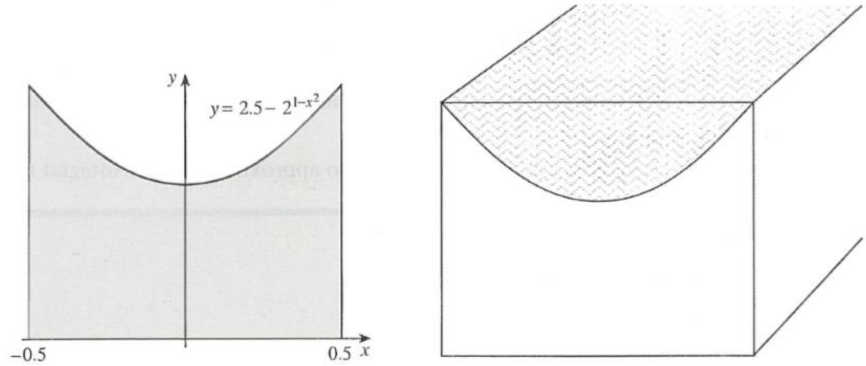
10 A river is 18 metres wide in a certain region and its depth, d metres, at a point x metres from one side is given by the formula $d = \frac{1}{18}\sqrt{x(18-x)(18+x)}$.

(a) Produce a table showing the depths (correct to 3 decimal places where necessary) at $x = 0,\ 3,\ 6,\ 9,\ 12,\ 15$ and 18.

(b) Use the trapezium rule to estimate the cross-sectional area of the river in this region.

(c) Given that, in this region, the river is flowing at a uniform speed of 100 metres per minute, estimate the number of cubic metres of water passing per minute. (OCR)

11 The left diagram shows the part of the curve $y = 2.5 - 2^{1-x^2}$ for which $-0.5 \leqslant x \leqslant 0.5$. The shaded region in the left diagram forms the cross-section of the straight concrete drainage channel shown in the right diagram. The units involved are metres.



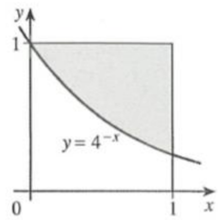
(a) Use the trapezium rule with 4 intervals to estimate the area of the shaded region.

(b) Estimate the volume of concrete in a 20-metre length of channel.

(c) Estimate the volume of water in the 20-metre length of channel when it is full.

(d) Of the estimates in parts (b) and (c), which is an overestimate and which is an underestimate?

12 The diagram shows the curve $y = 4^{-x}$. Taking subdivisions at $x = 0.25,\ 0.5,\ 0.75$, find an approximation to the shaded area.



13 The integral $\int_{36}^{64} \sqrt{x}\,dx$ is denoted by I.

(a) Find the exact value of I.

(b) Use the trapezium rule with 2 intervals to find an estimate for I, giving your answer in terms of $\sqrt{2}$.

Use your two answers to deduce that $\sqrt{2} \approx \frac{149}{105}$.

14* The trapezium rule, with 2 intervals of equal width, is to be used to find an approximate value for $\int_1^2 \frac{1}{x^2}\,dx$. Explain, with the aid of a sketch, why the approximation will be greater than the exact value of the integral.

Calculate the approximate value and the exact value, giving each answer correct to 3 decimal places.

Another approximation to $\int_1^2 \frac{1}{x^2}\,dx$ is to be calculated by using two trapezia of unequal width; the ordinates are at $x = 1$, $x = h$ and $x = 2$. Find, in terms of h, the total area, T, of these two trapezia.

Find the value of h for which T is a minimum, and find this minimum value of T, giving your answer correct to 3 decimal places.

15* (a) Calculate the exact value of the integral $\int_0^1 x^2\,dx$.

(b) Find the trapezium rule approximations to this integral using 1, 2, 4 and 8 intervals. Call these A_1, A_2, A_4 and A_8.

(c) For each of your answers in part (b), calculate the error E_i, where

$$E_i = \int_0^1 x^2\,dx - A_i, \quad \text{for} \quad i = 1,\ 2,\ 4 \text{ and } 8.$$

(d) Look at your results for part (c), and guess the relationship between the error E_n and the number n of intervals taken.

(e) How many intervals would you need to approximate to the integral to within 10^{-6}?

1 10.6

2 3.28; overestimate

3 1.701

4 0.52

5 0.70

6 51

7 (a) 1.41; the overestimate between 0 and 0.5 looks to be roughly balanced by the underestimate between 0 and −0.5.

8 1140 m

9 8.15 km

10 (a) 0, 1.708, 2.309, 2.598, 2.582, 2.141, 0
(b) 34.0 m^2 (c) 3400 m^3

11 (a) 0.622 m^2 (b) 12.4 m^3 (c) 3.9 m^3
(d) (b) overestimate, (c) underestimate

12 0.55

13 (a) $\frac{592}{3}$ (b) $98 + 70\sqrt{2}$

14 0.535, 0.5;
$T = \frac{3}{8}h - \frac{1}{4} + \frac{1}{2}h^{-2}$;
$h = 2 \times 3^{-\frac{1}{3}}$, or 1.39;
$T = \frac{3}{8} \times 3^{\frac{2}{3}} - \frac{1}{4} \approx 0.530$

15 (a) $\frac{1}{3}$ (b) $\frac{1}{2}, \frac{3}{8}, \frac{11}{32}, \frac{43}{128}$
(c) $-\frac{1}{6}, -\frac{1}{24}, -\frac{1}{96}, -\frac{1}{384}$
(d) $E_n = -\frac{1}{6n^2}$ (e) 409, or more