Circles (answers at the end)

1 Find the centre and radius of the circle with equation $x^2 + y^2 - 7x + y + 8 = 0$.

2 The circle with equation $x^2 + y^2 + 8x - 22y + c = 0$ has radius 7. Find the value of c.

3 The circle C has equation $x^2 + y^2 + 12x - 4y + 11 = 0$. Determine whether the following points lie inside, on or outside C.

$P(-4, 7)$ $Q(-5, -3)$ $R(-2, 6)$ $S(-1, 4)$ $T(-9, 6)$

4 Find the centre and radius of the circle with equation $3x^2 + 3y^2 - 9x + 15y + 23 = 0$.

5 Points A and B have coordinates $(-3, -6)$ and $(9, 2)$ respectively. Find the equation of the circle which has AB as diameter.

6 Find the centre and radius of the circle with equation $x^2 + y^2 - 6y = 0$. Find also the coordinates of the points of intersection of the line $x - 2y + 3 = 0$ and this circle. (OCR)

7 Find the equation of the tangent to the circle $x^2 + y^2 + 8x + 4y + 7 = 0$ at the point $(-1, 0)$.

8 Three points are $P(-2, 7)$, $Q(2, 3)$ and $R(4, 5)$. Show that PQ is perpendicular to QR. Find the equation of the circle which passes through the points P, Q and R.

9 The straight line $y = 20 - 3x$ meets the circle $x^2 + y^2 - 2x - 14y = 0$ at the points A and B. Calculate the exact length of the chord AB. (OCR)

10 The line $y = -3x + k$ is a tangent to the circle $x^2 + y^2 = 10$. Find the possible values of k.

11 The straight line $y = 2x + k$ meets the circle $x^2 + y^2 - 2x + 4y = 0$ at two points. Find the set of possible values of k.

12 (a) Determine the translation which transforms the circle with equation $x^2 + y^2 + 4x - 8y = 0$ to the circle with equation $x^2 + y^2 + 10x - 10y + 30 = 0$.

(b) The circle with equation $x^2 + y^2 - 7x - y - 3 = 0$ is translated 5 units in the positive x-direction and 2 units in the negative y-direction. Find the equation of the resulting circle.

(c) The circle with equation $x^2 + y^2 + 10x - 2y + 10 = 0$ is reflected in the x-axis and then translated by 4 units in the positive x-direction. Find the equation of the resulting circle.

13 Verify that the circle with equation $x^2 + y^2 - 2rx - 2ry + r^2 = 0$ touches both the coordinate axes. Find the radii of the two circles which pass through the point $(16, 2)$ and touch both the coordinate axes. (OCR)

14 It is given that the circle $x^2 + y^2 - 14x - 10y + c = 0$ lies wholly in the first quadrant. Show that $49 < c < 74$.

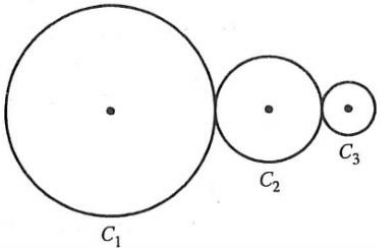
15 Prove that each of the circles $x^2 + y^2 - 4x = 0$ and $x^2 + y^2 - 12x - 8y + 43 = 0$ lies completely outside the other. (OCR)

16 Prove that the equation $x^2 + y^2 - 8x + 4ky + 3k^2 = 0$ represents a circle for all values of k.

17 Circle C_1 has equation $x^2 + y^2 + 4x - 6y - 12 = 0$ and circle C_2 has equation $x^2 + y^2 - 20x + 12y + 100 = 0$. Point P lies on C_1 and point Q lies on C_2. The distance between P and Q is denoted by d. Show that $4 \leqslant d \leqslant 26$.

18 A circle passes through the point $(9, -1)$ and is such that the straight lines $x = -7$ and $x = 13$ are tangents to the circle. Find the equation of each of the circles which satisfy these conditions.

19 The circles C_1, C_2 and C_3 touch as shown and have centres which lie on a straight line parallel to the x-axis. The radii are in the ratio 4 : 2 : 1. Given that the equation of C_1 is $x^2 + y^2 + 10x - 8y - 23 = 0$, find the equation of C_3.



1 $(\frac{7}{2}, -\frac{1}{2})$, $\frac{3}{2}\sqrt{2}$

2 88

3 on, inside, outside, on, inside

4 $(\frac{3}{2}, -\frac{5}{2})$, $\frac{1}{6}\sqrt{30}$

5 $x^2 + y^2 - 6x + 4y - 39 = 0$

6 $(0, 3)$, 3; $(-\frac{9}{5}, \frac{3}{5})$, $(3, 3)$

7 $3x + 2y + 3 = 0$

8 $x^2 + y^2 - 2x - 12y + 27 = 0$

9 $4\sqrt{10}$

10 ± 10

11 $-9 < k < 1$

12 (a) 3 units in the negative x-direction and 1 unit in the positive y-direction

(b) $x^2 + y^2 - 17x + 3y + 59 = 0$

(c) $x^2 + y^2 + 2x + 2y - 14 = 0$

13 10, 26

18 $x^2 + y^2 - 6x + 18y - 10 = 0$,
$x^2 + y^2 - 6x - 14y - 42 = 0$

19 $x^2 + y^2 - 26x - 8y + 181 = 0$