Quadratic functions (answers at the end)

# Mixed practice 3

In this exercise, you must show detailed reasoning.

1. A quadratic function passes through the points $(k, 0)$ and $(k+4, 0)$. Find the $x$-coordinate of the vertex of the graph of the function.

2. Solve algebraically:

   $(2x-3)(x-5)=(x-3)^2$

3. Solve $x^4-5x^2+4=0$.

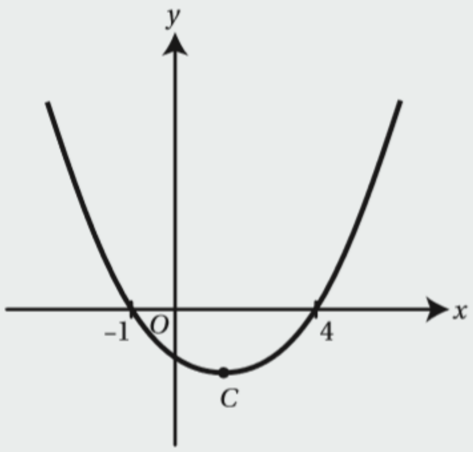
4. The quadratic function $y=(x-a)^2+b$ has a turning point at $(3, 7)$.

   **a** State whether this turning point is a maximum or a minimum point.

   **b** State the values of $a$ and $b$.

5. The quadratic function $y=a(x-b)^2+c$ passes through the points $(-2, 0)$ and $(6, 0)$. Its maximum $y$ value is 48. Find the values of $a$, $b$ and $c$.

6. The diagram represents the graph of the function $f(x)=(x+p)(x-q)$.




   **a** Write down the values of $p$ and $q$ if they are both positive.

   **b** The function has a minimum value at the point $C$. Find the $x$-coordinate of $C$.

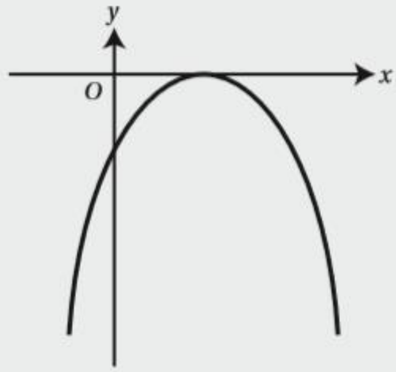
7. **i** Find the discriminant of $kx^2-4x+k$ in terms of $k$.

   **ii** The quadratic equation $kx^2-4x+k=0$ has equal roots. Find the possible values of $k$.



8. Solve simultaneously $x^2-2x>0$ and $x^2-4x+3\geqslant 0$.

**9** The diagram shows the graph of the function $y = ax^2 + bx + c$.




Copy and complete this table to show whether each expression is positive, negative or zero.

| Expression | Positive | Negative | Zero |
|---|---|---|---|
| $a$ | | | |
| $c$ | | | |
| $b^2 - 4ac$ | | | |
| $b$ | | | |

**10** a Write $x^2 - 10x + 35$ in the form $(x - p)^2 + q$.

b Hence, or otherwise, find the maximum value of $\frac{1}{(x^2 - 10x + 35)^3}$.

**11** Find the exact values of $k$ for which the equation $2kx^2 + (k + 1)x + 1 = 0$ has no real roots.

**12** Solve the equation: $x^{\frac{1}{4}} + 2x^{-\frac{1}{4}} = 3$.

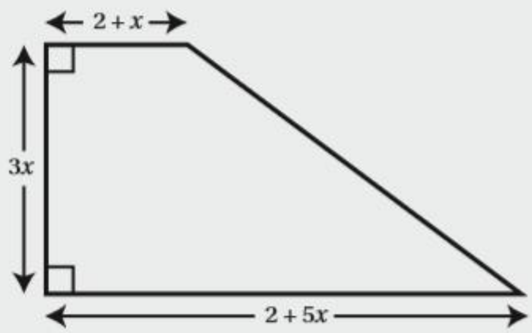
**13** Solve the equation $\frac{49}{(5x+2)^2} - \frac{14}{5x+2} + 1 = 0$.

**14** a Express $2x^2 - 6x + 9$ in the form $p(x + q)^2 + r$.

b State the coordinates of the vertex of the curve $y = 2x^2 - 6x + 9$.

c State the number of real roots of the equation $2x^2 - 6x + 9 = 0$.

**15** A lawn is to be made in the shape shown. The units are metres.




i The perimeter of the lawn is $P$ m. Find $P$ in terms of $x$.

ii Show that the area, $A$ m², of the lawn is given by $A = 9x^2 + 6x$.

The perimeter of the lawn must be at least 39 m and the area of the lawn must be less than 99 $m^2$.

**iii** By writing down and solving appropriate inequalities, determine the set of possible values of $x$.



**16** Alexia and Michaela were both trying to solve a quadratic equation of the form $x^2 + bx + c = 0$.

Unfortunately Alexia misread the value of $b$ and found that the solutions were 6 and 1.

Michaela misread the value of $c$ and found that the solutions were 4 and 1.

What were the correct solutions?

**17** Find the values of $k$ for which the line $y = 2x - k$ is tangent to the curve with equation $x^2 + y^2 = 5$.

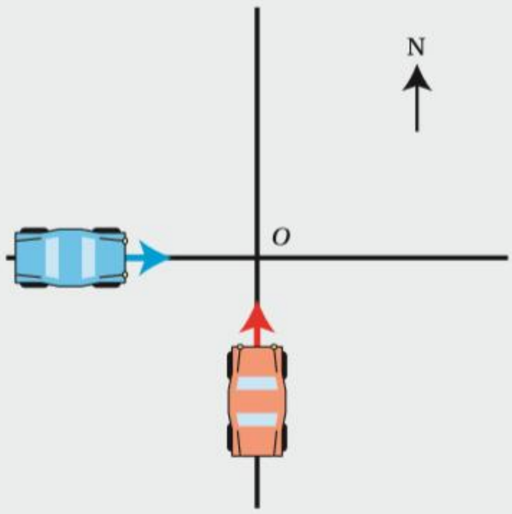
**18** Let $\alpha$ and $\beta$ denote the roots of the quadratic equation $x^2 - kx + (k - 1) = 0$.

**a** Express $\alpha$ and $\beta$ in terms of the real parameter $k$.

**b** Given that $\alpha^2 + \beta^2 = 17$, find the possible values of $k$.

**19** Let $q(x) = kx^2 + (k - 2)x - 2$. Show that the equation $q(x) = 0$ has real roots for all values of $k$.

**20** Two cars are travelling along two straight roads that are perpendicular to each other and meet at the point $O$, as shown in the diagram. The first car starts 50 km west of $O$ and travels east at the constant speed of 20 km/h. At the same time, the second car starts 30 km south of $O$ and travels north at the constant speed of 15 km/h.




**a** Show that at time $t$ (hours) the distance $d$ (km) between the two cars satisfies
$d^2 = 625t^2 - 2900t + 3400$.

**b** Hence find the closest distance between the two cars.

**1** $x = k + 2$

**2** $x = 1, 6$

**3** $x = \pm 1, \pm 2$

**4** **a** Minimum **b** $a = 3, b = 7$

**5** $a = -3, b = 2, c = 48$

**6** **a** $p = 1, q = 4$ **b** $x = 1.5$

**7** **a** $16 - 4k^2$ **b** $k = \pm 2$ or $-2$

**8** $x \geqslant 3$ or $x < 0$

**9** $a$, $c$ negative, $b$ positive, $b^2 - 4ac = 0$

**10** **a** $(x - 5)^2 + 10$ **b** $\frac{1}{1000}$

**11** $3 - 2\sqrt{2} < k < 3 + 2\sqrt{2}$

**12** $x = 1, 16$

**13** $x = 1$

**14** **a** $2\left(x - \frac{3}{2}\right)^2 + \frac{9}{2}$ **b** $\left(\frac{3}{2}, \frac{9}{2}\right)$ **c** Zero

**15** **a** $P = 14x + 4$ **b** Proof **c** $\frac{5}{2} \leqslant x < 3$

**16** $x = 3, 2$

**17** $k = \pm 5$

**18** **a** $k - 1, 1$ **b** $k = -3, 5$

**19** Proof

**20** **a** Proof **b** 6 km