Exponentials and logarithms (answers at the end)

1 Solve each of the following equations to find x in terms of a where $a > 0$ and $a \neq 100$. The logarithms are to base 10.

(a) $a^x = 10^{2x+1}$

(b) $2\log(2x) = 1 + \log a$ (OCR, adapted)

2 Solve the equation $3^{2x} = 4^{2-x}$, giving your answer to three significant figures. (OCR)

3 Find y in terms of x for each of the following equations.

(a) $3^y = 5^x$, expressing your answer in the form $y = cx$ and giving the exact value of the constant c.

(b) $\ln(x^4) - \ln(x^2y) + \ln(y^2) = 0$, expressing your answer in the form $y = kx^n$ and giving the values of the constants k and n.

4 Find the root of the equation $10^{2-2x} = 2 \times 10^{-x}$ giving your answer exactly in terms of logarithms. - (OCR, adapted)

5 Given the simultaneous equations

$$2^x = 3^y,$$
$$x + y = 1,$$

show that $x = \dfrac{\log 3}{\log 6}$. (OCR, adapted)

6 Express $\log(2\sqrt{10}) - \frac{1}{3}\log 0.8 - \log\left(\frac{10}{3}\right)$ in the form $c + \log d$ where c and d are rational numbers and the logarithms are to base 10. (OCR, adapted)

7 An athlete plans a training schedule which involves running 20 km in the first week of training; in each subsequent week the distance is to be increased by 10% over the previous week. Write down an expression for the distance to be covered in the nth week according to this schedule, and find in which week the athlete would first cover more than 100 km.

8 Prove that $\log\left(\dfrac{p}{q}\right) + \log\left(\dfrac{q}{r}\right) + \log\left(\dfrac{r}{p}\right) = 0$.

9 If a, b and c are positive numbers in geometric progression, show that $\log a$, $\log b$ and $\log c$ are in arithmetic progression.

10 Express $\log_2(x+2) - \log_2 x$ as a single logarithm.
Hence solve the equation $\log_2(x+2) - \log_2 x = 3$.

11 Solve the inequality $8^x > 10^{30}$. (OCR)

12 (a) A curve has equation $y = 12 \times 4^x$. Find the value of x for which $y = 20\,000$.

(b) The graph of $y = 12 \times 4^x$ is translated by one unit parallel to the positive x-axis. Given that the new graph has equation $y = cd^x$, write down the values of c and d.

(c) The graph of $y = 12 \times 4^x$ is transformed by a stretch of scale factor 2 parallel to the x-axis followed by a stretch of scale factor $\frac{1}{3}$ parallel to the y-axis. Given that the new graph has equation $y = gh^x$, find the values of g and h. (MEI)

13 Given that $\log_e p = 2600$, find the values of

(a) $\log_e(pe^{10})$, (b) $\log_e\left(\dfrac{1}{\sqrt{p}}\right)$. (OCR, adapted)

14 (a) Solve the equation $\log_e\left(\frac{1}{2}x + 1\right) = 8$, giving your answer in terms of e.

(b) Solve the equation $e^{\frac{1}{2}x+1} = 8$, giving your answer in terms of $\log_e 2$. (OCR, adapted)

15* If $\log_r p = q$ and $\log_q r = p$ express p in terms of q and r, and r in terms of p and q. Hence prove that $\log_q p = pq$.

16* If $\log_b a = p$, $\log_c b = q$ and $\log_a c = r$, where a, b and c are positive numbers, express a in terms of b and p, b in terms of c and q, and c in terms of a and r. Deduce that $a^{pqr} = a$, and hence prove that $\log_b a \times \log_c b \times \log_a c = 1$.

1 (a) $\dfrac{1}{\log a - 2}$ (b) $\sqrt{\dfrac{5a}{2}}$

2 0.774

3 (a) $y = \dfrac{\log 5}{\log 3}x$; $c = \dfrac{\log 5}{\log 3}$

(b) $y = x^{-2}$; $k = 1$, $n = -2$

4 $2 - \log 2$

6 $\log 3 - \frac{1}{6}$

7 $20 \times 1.1^{n-1}$; 18

10 $\log_2 \dfrac{x+2}{x}$, $\frac{2}{7}$

11 $x > 33.2$

12 (a) 5.35 (b) 3, 4 (c) 4, 2

13 (a) 2610 (b) -1300

14 (a) $x = 2(e^8 - 1)$ (b) $x = 6\log_e 2 - 2$

15 $p = r^q$, $r = q^p$

16 $a = b^p$, $b = c^q$, $c = a^r$