Quadratics (answers at the end)

1 Solve the simultaneous equations $x + y = 2$ and $x^2 + 2y^2 = 11$. (OCR)

2 The quadratic polynomial $x^2 - 10x + 17$ is denoted by $f(x)$. Express $f(x)$ in the form $(x - a)^2 + b$ stating the values of a and b.

Hence find the least possible value that $f(x)$ can take and the corresponding value of x. (OCR)

3 Solve the simultaneous equations $2x + y = 3$ and $2x^2 - xy = 10$. (OCR)

4 For what values of k does the equation $2x^2 - kx + 8 = 0$ have a repeated root?

5 (a) Solve the equation $x^2 - (6\sqrt{3})x + 24 = 0$, giving your answer in terms of surds, simplified as far as possible.

(b) Find all four solutions of the equation $x^4 - (6\sqrt{3})x^2 + 24 = 0$ giving your answers correct to 2 decimal places. (OCR)

6 Show that the line $y = 3x - 3$ and the curve $y = (3x + 1)(x + 2)$ do not meet.

7 Express $9x^2 - 36x + 52$ in the form $(Ax - B)^2 + C$, where A, B and C are integers. Hence, or otherwise, find the set of values taken by $9x^2 - 36x + 52$ for real x. (OCR)

8* Find the points of intersection of the curves $y = 6x^2 + 4x - 3$ and $y = x^2 - 3x - 1$, giving the coordinates correct to 2 decimal places.

9 (a) Express $9x^2 + 12x + 7$ in the form $(ax + b)^2 + c$ where a, b, c are constants whose values are to be found.

(b) Find the set of values taken by $\dfrac{1}{9x^2 + 12x + 7}$ for real values of x. (OCR)

10 Find, correct to 3 significant figures, all the roots of the equation $8x^4 - 8x^2 + 1 = \frac{1}{2}\sqrt{3}$. (OCR)

11 Find constants a, b and c such that, for all values of x,

$$3x^2 - 5x + 1 = a(x + b)^2 + c.$$

Hence find the coordinates of the minimum point on the graph of $y = 3x^2 - 5x + 1$.
(Note: the minimum point or maximum point is the vertex.) (OCR, adapted)

12 Find the points of intersection of the curve $xy = 6$ and the line $y = 9 - 3x$. (OCR)

13* The equation of a curve is $y = ax^2 - 2bx + c$, where a, b and c are constants with $a > 0$.

(a) Find, in terms of a, b and c, the coordinates of the vertex of the curve.

(b) Given that the vertex of the curve lies on the line $y = x$, find an expression for c in terms of a and b. Show that in this case, whatever the value of b, $c \geqslant -\dfrac{1}{4a}$. (OCR, adapted)

14 (a) The diagram shows the graphs of $y = x - 1$ and $y = kx^2$, where k is a positive constant. The graphs intersect at two distinct points A and B. Write down the quadratic equation satisfied by the x-coordinates of A and B, and hence show that $k < \frac{1}{4}$.

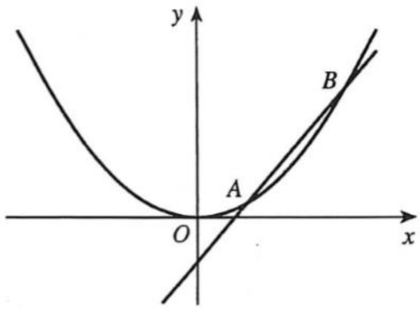

Fig. 4.4

(b) Describe briefly the relationship between the graphs of $y = x - 1$ and $y = kx^2$ in each of the cases (i) $k = \frac{1}{4}$, (ii) $k > \frac{1}{4}$.

(c) Show, by means of a graphical argument or otherwise, that when k is a negative constant, the equation $x - 1 = kx^2$ has two real roots, one of which lies between 0 and 1.

15 Use the following procedure to find the least (perpendicular) distance of the point $(1, 2)$ from the line $y = 3x + 5$, *without* having to find the equation of a line perpendicular to $y = 3x + 5$ (as you did in Chapter 1).

(a) Let (x, y) be a general point on the line. Show that its distance, d, from $(1, 2)$ is given by $d^2 = (x - 1)^2 + (y - 2)^2$.

(b) Use the equation of the line to show that $d^2 = (x - 1)^2 + (3x + 3)^2$.

(c) Show that $d^2 = 10x^2 + 16x + 10$.

(d) By completing the square, show that the minimum distance required is $\frac{3}{5}\sqrt{10}$.

16 Point O is the intersection of two roads which cross at right angles; one road runs from north to south, the other from east to west. Car A is 100 metres due west of O and travelling east at a speed of 20 m s^{-1}, and Car B is 80 metres due north of O and travelling south at 20 m s^{-1}.

(a) Show that after t seconds their distance apart, d metres, is given by $d^2 = (100 - 20t)^2 + (80 - 20t)^2$.

(b) Show that this simplifies to $d^2 = 400((5 - t)^2 + (4 - t)^2)$.

(c) Show that the minimum distance apart of the two cars is $10\sqrt{2}$ metres.

17 A mail-order photographic developing company offers a picture-framing service to its customers. It will enlarge and mount any photograph, under glass and in a rectangular frame. Its charge is based on the size of the enlargement. It charges £6 per metre of perimeter for the frame and £15 per square metre for the glass. Write down an expression for the cost of enlarging and mounting a photograph in a frame which is x metres wide and y metres high.

A photograph was enlarged and mounted in a square frame of side z metres at a cost of £12. Formulate and solve a quadratic equation for z.

18 (a) Calculate the discriminant of the quadratic polynomial $2x^2 + 6x + 7$.

(b) State the number of real roots of the equation $2x^2 + 6x + 7 = 0$, and hence explain why $2x^2 + 6x + 7$ is always positive.

19 Solve the simultaneous equations $y = 2x^2 - 3x + 4$, $y = 4x + 1$. (OCR)

20 (a) Express $4x^2 - 16x + 8$ in the form $a(x+b)^2 + c$.

(b) Hence find the coordinates of the vertex of the graph of $y = 4x^2 - 16x + 8$.

(c) Sketch the graph of $y = 4x^2 - 16x + 8$, giving the x-coordinates of the points where the graph meets the x-axis. (OCR)

21 It is given that x and y satisfy the simultaneous equations $y - x = 4$, $2x^2 + xy + y^2 = 8$.

(a) Show that $x^2 + 3x + 2 = 0$.

(b) Solve the simultaneous equations. (OCR)

22 (a) Calculate the discriminant of $3x^2 - 4x + 2$.

(b) Hence state the number of real roots of the equation $3x^2 - 4x + 2 = 0$. (OCR)

23 (a) Express $x^2 + 8x + 18$ in the form $(x+a)^2 + b$.

(b) Sketch the graph of $y = x^2 + 8x + 18$, stating the coordinates of its vertex. (OCR)

24 (a) Given that $\sqrt{x} = y$, show that the equation $\sqrt{x} + \dfrac{10}{\sqrt{x}} = 7$ may be written as $y^2 - 7y + 10 = 0$.

(b) Hence solve the equation $\sqrt{x} + \dfrac{10}{\sqrt{x}} = 7$. (OCR)

1 $x = 3,\ y = -1$ or $x = -\frac{1}{3},\ y = \frac{7}{3}$

2 $a = 5,\ b = -8$. Least value is -8 when $x = 5$.

3 $x = 2,\ y = -1$ or $x = -\frac{5}{4},\ y = \frac{11}{2}$

4 8 and -8

5 (a) $2\sqrt{3}, 4\sqrt{3}$ (b) $\pm 1.86,\ \pm 2.63$

7 $(3x-6)^2 + 16$. Takes values $\geqslant 16$.

8 $(-1.64, 6.63)$, $(0.24, -1.67)$

9 (a) $(3x+2)^2 + 3$
(b) $0 < f(x) \leqslant \frac{1}{3}$

10 ± 0.991 and ± 0.131

11 $a = 3,\ b = -\frac{5}{6},\ c = -\frac{13}{12}$.
Minimum is $\left(\frac{5}{6}, -\frac{13}{12}\right)$.

12 $(1, 6)$ and $(2, 3)$

13 (a) $(b/a, c - b^2/a)$ (b) $c = b(b+1)/a$

14 (a) $kx^2 - x + 1 = 0$
(b) (i) Line is tangent to curve.
(ii) Line and curve do not intersect.

17 £$(12(x+y) + 15xy)$; $5z^2 + 8z - 4 = 0$; 0.4

18 (a) -20
(b) None; so the graph never crosses the x-axis and is either always positive or always negative. When $x = 0$, $2x^2 + 6x + 7 = 7$. As $7 > 0$, $2x^2 + 6x + 7$ is always positive.

19 $x = \frac{1}{2},\ y = 3$ or $x = 3,\ y = 13$

20 (a) $4(x-2)^2 - 8$ (b) $(2, -8)$
(c) $2 - \sqrt{2},\ 2 + \sqrt{2}$

21 (b) $x = -2,\ y = 2$ or $x = -1,\ y = 3$

22 (a) -8 (b) None

23 (a) $(x+4)^2 + 2$ (b) $(-4, 2)$

24 (b) 4, 25