Mixed practice 8

1. **a** Sketch the graph of $y = e^{0.8x}$.

   b Find the gradient of your graph at the point where $x = 3$.

   c Use your graph to determine the number of solutions of the equation $e^{0.8x} = \frac{1}{x}$.

2. The amount of substance in a chemical reaction is decreasing according to the equation $m = 32e^{-0.14t}$ where m grams is the mass of the substance t seconds after the start of the reaction.

   a State the amount of the substance at the start of the reaction.

   b At what rate is the amount of substance decreasing 3 seconds after the start of the reaction?

   c How long will it take for the amount of substance to halve?

3. Use graphs to determine the number of solutions of the equation $\ln x = \frac{3}{x^2}$.

4. The volume of a blob of algae (V) in cm^3 in a jar is modelled by $V = 0.4 \times 2^{0.1t}$ where t is the time in weeks after the observation begins.

   a What is the initial volume of the algae?

   b How long does it take for the volume of algae to double?

   c Give two reasons why the model would not be valid for predicting the volume in 10 years' time.

5. A rumour spreads exponentially through a school. When school begins (at 9 a.m.) 18 people know it.
   By 10 a.m. 42 people know it.

   Let N be the number of people who know the rumour after t minutes.

   a Find constants A and k so that $N = Ae^{kt}$.

   b How many people know the rumour at 10:30?

   c There are 1200 people in the school. According to the exponential model at what time will everyone know the rumour?

6. A patient is being treated for a condition by having insulin injected. The level of insulin (I) in the blood t minutes after the injection is given by $I = 10e^{-0.05t} + 2$, measured in microunits per millilitre (μU/ml).

   a What is the level of insulin immediately after the injection?

   b There is a danger of coma if insulin levels fall below 1.8 μU/ml. According to the model, will this level be reached? Justify your answer.

7 It is thought that the global population of tigers is falling exponentially. Estimates suggest that in 1970 there were 37 000 tigers but by 1980 the number had dropped to 22 000.

a A model of the form $T = ka^n$ is suggested, connecting the number of tigers (T) with the number of years (n) after 1970.

i Show that $22\,000 = ka^{10}$.

ii Write another similar equation and solve them to find k and a.

b What does the model predict the tiger population will be in 2020?

c When the population reaches 1000, the tiger population will be described as 'near extinction'. In which year will this happen?

8 A zoologist believes that the population of fish in a small lake is growing exponentially. He collects data about the number of fish every 10 days for 50 days. The data are given in this table:

Time (days)	0	10	20	30	40	50
Number of fish	35	42	46	51	62	71

The zoologist proposes a model of the form $N = Ae^{kt}$ where N is the number of fish and t is time in days. In order to estimate the values of the constant A and k he plots a graph with t on the horizontal axis and $\ln N$ on the vertical axis.

a Explain why, assuming the zoologist's model is correct, this graph will be approximately a straight line.

b Complete the table of values for the graph:

t	0	10	20	30	40	50
$\ln N$	3.56	3.74	3.83	3.93		4.26

c Find the equation of the line of best fit for this table. (Do not draw the graph.) Hence estimate the values of A and k.

d Use this model to predict the number of fish in the lake when $t = 260$.

e The zoologist finds that the number of fish in the lake after 260 days is actually 720. Suggest one reason why the observed data does not fit the prediction.

9 Quantities m and t are related by an equation of the form $m = at^p$ where a and p are constants. The graph of $\log m$ against $\log t$ is a straight line that passes through the points (2, 5) and (4, 0). Find the values of a and p.

10 A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula $m = 240e^{-0.04t}$.

i Find the time taken for the substance to halve its mass.

ii Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year.

11 The mass, M grams, of a certain substance is increasing exponentially so that, at time t hours, the mass is given by $M = 40e^{kt}$, where k is a constant. The following table shows certain values of t and M.

t	0	21	63
M		80	

i In either order,

a find the values missing from the table,

b determine the value of k.

ii Find the rate at which the mass is increasing when $t = 21$.

12 Radioactive decay can be modelled using an equation of the form $m = m_0e^{-kt}$ where m is the mass of the radioactive substance at time t, m_0 is the initial mass and k is a positive constant. The half-life of a radioactive substance is the length of time it takes for half of the substance to decay. A particular radioactive substance has a half-life of 260 years. Find the value of k.

13 The speed, V m s^{-1}, of a parachutist, t seconds after jumping from the aeroplane, is modelled by the equation $V = 42(1 - e^{-0.2t})$.

a What is the initial speed of the parachutist?

b What is the maximum speed that the parachutist could reach?

c When the parachutist reaches 22 m s^{-1} he opens the parachute. For how long is he falling before he opens his parachute?

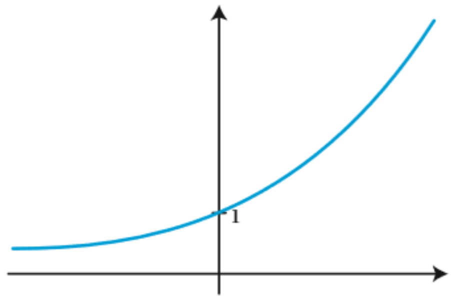
14 When a cup of tea is first made its temperature is 98 °C. After two minutes the temperature has reached 94 °C. The room temperature is 22 °C and the difference between the temperature of the tea and the room temperature decreases exponentially.

a Let T be the temperature of the tea and t be the time, in minutes, since the tea was made. Find the constants C and t so that $T - 22 = Ce^{-kt}$.

b Find the time it takes for the tea to cool to 78 °C.

Mixed practice 8

1 a



b 8.82 **c** One

2 a 32 g **b** $-2.94\ \mathrm{g\ s^{-1}}$ **c** 4.95 s

3 One

4 a $0.4\ \mathrm{cm^3}$ **b** 10 weeks

c Model is for algae in a jar, which limits volume; extrapolation beyond model's validity

5 a $A = 18$, $k = 0.0141$ **b** 64

c 1.58 p.m.

6 a 12 μU/ml

b No – long-term level is 2 μU/ml

7 a i Proof

ii $k = 37\,000$, $a = 0.949$

b 2700 **c** 2039

8 a $\ln(N) = kt + \ln(A)$ **b** 4.13, 4.26

c $\ln(N) = 0.0137t + 3.56$; $N = 35.2\mathrm{e}^{0.0137t}$

d 1240

e Size of the lake limits indefinite growth; seasonal variation

9 $p = -2.5$, $a = 10^{10}$

10 a 17.3 years **b** 38.0 years

11 a i 40, 320 **ii** $k = 0.0330$

b $2.4\mathrm{g\ h^{-1}}$

12 0.002 67 s

13 a $0\ \mathrm{m\ s^{-1}}$ **b** $42\ \mathrm{m\ s^{-1}}$ **c** 3.71 s

14 a $C = 76$, $k = 0.027$ **b** 11.3 min