Surds (answers at the end)

**1** Simplify the following.

(a) $5(\sqrt{2}+1)-\sqrt{2}(4-3\sqrt{2})$ (b) $(\sqrt{2})^4+(\sqrt{3})^4+(\sqrt{4})^4$

(c) $(\sqrt{5}-2)^2+(\sqrt{5}-2)(\sqrt{5}+2)$ (d) $(2\sqrt{2})^5$.

**2** Simplify the following.

(a) $\sqrt{27}+\sqrt{12}-\sqrt{3}$ (b) $\sqrt{63}-\sqrt{28}$

(c) $\sqrt{100\,000}+\sqrt{1000}+\sqrt{10}$ (d) $\sqrt[3]{2}+\sqrt[3]{16}$.

**3** Rationalise the denominators of the following.

(a) $\dfrac{9}{2\sqrt{3}}$ (b) $\dfrac{1}{5\sqrt{5}}$ (c) $\dfrac{2\sqrt{5}}{3\sqrt{10}}$ (d) $\dfrac{\sqrt{8}}{\sqrt{15}}$

**4** Simplify the following.

(a) $\dfrac{4}{\sqrt{2}}-\dfrac{4}{\sqrt{8}}$ (b) $\dfrac{10}{\sqrt{5}}+\sqrt{20}$

(c) $\dfrac{1}{\sqrt{2}}(2\sqrt{2}-1)+\sqrt{2}(1-\sqrt{8})$ (d) $\dfrac{\sqrt{6}}{\sqrt{2}}+\dfrac{3}{\sqrt{3}}+\dfrac{\sqrt{15}}{\sqrt{5}}+\dfrac{\sqrt{18}}{\sqrt{6}}$.

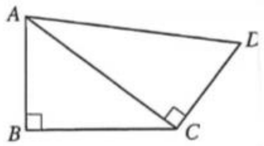
**5** Rationalise the denominators of the following.

(a) $\dfrac{4}{3-\sqrt{3}}$ (b) $\dfrac{6}{5+\sqrt{5}}$ (c) $\dfrac{3-\sqrt{2}}{3+\sqrt{2}}$ (d) $\dfrac{2\sqrt{7}-3}{4+\sqrt{7}}$

**6** Express $\dfrac{5}{\sqrt{7}}$ in the form $k\sqrt{7}$ where $k$ is a rational number. (OCR

**7** Find the gradient of the line joining $(1, 2)$ to $(\sqrt{2}, 3)$.

**8** In the diagram, angles $ABC$ and $ACD$ are right angles. Given that $AB = CD = 2\sqrt{6}$ cm and $BC = 7$ cm, show that the length of $AD$ is between $4\sqrt{6}$ cm and $7\sqrt{2}$ cm.




**9** In the triangle $PQR$, $Q$ is a right angle, $PQ = (6-2\sqrt{2})$ cm and $QR = (6+2\sqrt{2})$ cm.

(a) Find the area of the triangle. (b) Show that the length of $PR$ is $2\sqrt{22}$ cm.

**10** It can be shown that $\tan 75^\circ = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. Use a calculator to check this, and write an expression for $\tan 75^\circ$ in the form $a+b\sqrt{3}$, where $a$ and $b$ are rational numbers.

**11** Solve the simultaneous equations $5x-3y=41$ and $(7\sqrt{2})x+(4\sqrt{2})y=82$.

**12** The coordinates of the points $A$ and $B$ are $(2, 3)$ and $(4, -3)$ respectively. Find the length of $AB$ and the coordinates of the mid-point of $AB$. (OCR)

**13** An isosceles right-angled triangle has its two shorter sides of length $a$. Write down an expression for its perimeter in terms of $a$.

A length of rope 10 metres long is to be pegged out to form an isosceles right-angled triangle. Find, in as simple a form as possible, exact expressions for the lengths of the sides.

**14** (a) Find the equation of the line $l$ through the point $A(2, 3)$ with gradient $-\frac{1}{2}$.

(b) Show that the point $P$ with coordinates $(2+2t, 3-t)$ will always lie on $l$ whatever the value of $t$.

(c) Find the values of $t$ such that the length $AP$ is 5 units.

(d) Find the value of $t$ such that $OP$ is perpendicular to $l$ (where $O$ is the origin). Hence find the length of the perpendicular from $O$ to $l$.

**15** You are given that $y$ is not 0, and that $x > y$. Now suppose that $\sqrt{x-y}=\sqrt{x}-\sqrt{y}$.

(a) Show that $(\sqrt{x}-\sqrt{y})^2 = x-2\sqrt{x}\sqrt{y}+y$.

(b) Deduce that $y(x-y)=0$, and hence that either $y=0$ or $x=y$.

(c) What can you deduce about $\sqrt{x-y}$ and $\sqrt{x}-\sqrt{y}$?

**1** (a) $11+\sqrt{2}$ (b) $29$
(c) $10-4\sqrt{5}$ (d) $128\sqrt{2}$

**2** (a) $4\sqrt{3}$ (b) $\sqrt{7}$
(c) $111\sqrt{10}$ (d) $3\sqrt[3]{2}$

**3** (a) $\frac{3}{2}\sqrt{3}$ (b) $\frac{1}{25}\sqrt{5}$
(c) $\frac{1}{3}\sqrt{2}$ (d) $\frac{2}{15}\sqrt{30}$

**4** (a) $\sqrt{2}$ (b) $4\sqrt{5}$
(c) $-2+\frac{1}{2}\sqrt{2}$ (d) $4\sqrt{3}$

**5** (a) $\sqrt{7}+\sqrt{3}$ (b) $\frac{15-3\sqrt{5}}{10}$
(c) $\frac{11-6\sqrt{2}}{7}$ (d) $\frac{11\sqrt{7}-26}{9}$

**6** $\frac{5}{7}\sqrt{7}$

**7** $\sqrt{2}+1$

**9** (a) $14\,\text{cm}^2$

**10** $2+\sqrt{3}$

**11** $x=3\sqrt{2}+4$, $y=5\sqrt{2}-7$

**12** $2\sqrt{10}$, $(3, 0)$

**13** $a(2+\sqrt{2})$; $5(2-\sqrt{2})$ m, $5(2-\sqrt{2})$ m, $10(\sqrt{2}-1)$ m

**14** (a) $x+2y=8$ (c) $\sqrt{5}$ or $-\sqrt{5}$
(d) $t=-\frac{1}{5}$, $\frac{8}{5}\sqrt{5}$

**15** (c) They cannot be equal.