Mixed practice 13

1. Find the gradient function of $f(x)=\frac{3x-2}{\sqrt{x}}$.

2. A curve has equation $y=(4x^2-1)(3-x)$. Find $\frac{dy}{dx}$.

3. $f(x)=\frac{x^2-4}{2x}$. Find $f''(2)$.

4. Given that $f(x)=3\sqrt{x}-\frac{2}{\sqrt{x}}$ find:

   a $f'(x)$

   b the gradient of the graph of $y=f(x)$ at the point where $x=4$.

5. $f(x)=x^2+bx+c$. If $f(1)=2$ and $f'(2)=12$ find the values of b and c.

6. **a** Find the gradient of the curve $y=3\sqrt{x}-2$ at the point where it crosses the x-axis.

   b Is the curve increasing or decreasing at this point? Give a reason for your answer.

7. Find the range of values of x for which the function $y=3x^2-4x$ is increasing.

8. Find the rate of change of gradient of $y=x^2-2\sqrt{x}$ at the point where $x=9$.

9. Given that $y=\frac{5}{x^2}-\frac{1}{4x}+x$, find:

   i $\frac{dy}{dx}$, **ii** $\frac{d^2y}{dx^2}$.

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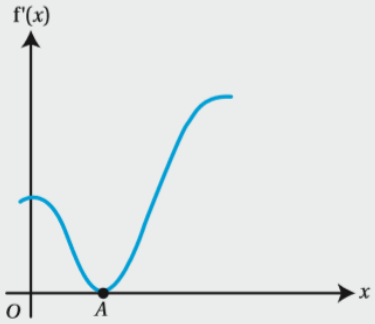
10. $y=x^2+ax-7$ is increasing for $x>5$. Find a.

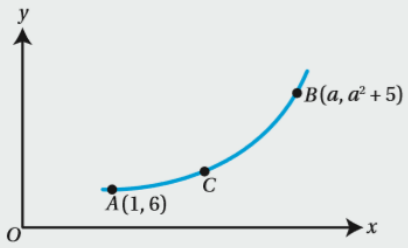
11. What is the rate of change of the gradient of $y=x^3+4x^2-2x+1$ at $x=\frac{1}{2}$?

12. This graph shows the gradient function, $f'(x)$, of a function $f(x)$.
    Which of the following is definitely true at the point A?



A $f(x)$ has a minimum **B** $f(x)$ has a maximum **C** $f(x)=0$ **D** $f''(x)=0$

13. The diagram shows part of the curve $y=x^2+5$. The point A has coordinates $(1, 6)$. The point B has coordinates (a, a^2+5), where a is a constant greater than 1. The point C is on the curve between A and B.



i Find by differentiation the value of the gradient of the curve at the point A.

ii The line segment joining the points A and B has gradient 2.3. Find the value of a.

iii State a possible value for the gradient of the line segment joining the points A and C.

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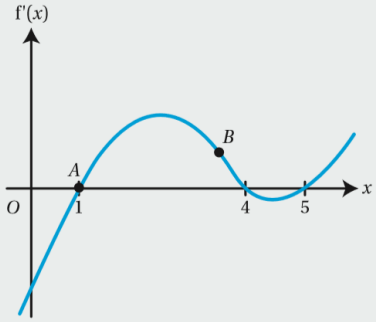
14 Use differentiation from first principles to find $\frac{dy}{dx}$ for $y = x^3 - 5x$.

15 The diagram shows the graph of $y = f'(x)$.



a State the value of the gradient of the graph of $y = f(x)$ at the point marked A.

b Is the function $f(x)$ increasing or decreasing at the point marked B?

c Sketch the graph of $y = f(x)$.

16 Find the coordinates of the point on the curve $y = \sqrt{x} + 3x$ where the gradient is 5.

17 Find the gradient of the graph of $y = \frac{1}{2\sqrt{x}}$ at the point where the y-coordinate is 3.

18 $f(x) = ax^3 + bx^{-2}$ where a and b are constants. $f'(1) = 18$ and $f''(1) = 18$.
Find a and b.

19 $f(x) = \sqrt{x^3} + 15\sqrt{x}$
Find the values of x for which the gradient of $f(x)$ is 9.

20 Find the range of values of x for which the gradient of the graph $y = x^4 - 2x^2 + 3$ is decreasing.

Mixed practice 13

1 $\frac{3}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$

2 $\frac{dy}{dx} = -12x^2 + 24x + 1$

3 $-\frac{1}{2}$

4 **a** $f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$ **b** $\frac{7}{8}$

5 $b = 8$, $c = -7$

6 **a** $\frac{9}{4}$

b Positive gradient; increasing

7 $x > \frac{2}{3}$

8 $\frac{109}{54}$

9 **a** $-\frac{10}{x^3} + \frac{1}{4x^2} + 1$ **b** $\frac{30}{x^4} - \frac{1}{2x^3}$

10 $a = -10$

11 $\frac{11}{4}$

12 D

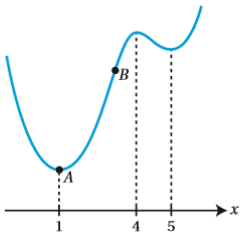
13 **a** 2 **b** 1.3

c Anything between 2 and 2.3

14 $3x^2 - 5$

15 **a** 0 **b** Increasing

c



16 $\left(\frac{1}{16}, \frac{7}{16}\right)$

17 -54

18 $a = 4.8$, $b = -1.8$

19 $x = 1, 25$

20 $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$