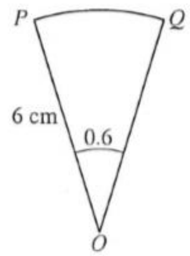
Radians (answers at the end)

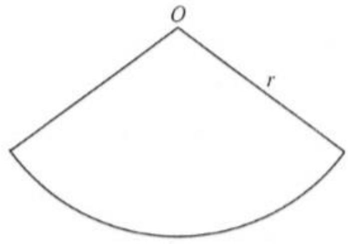
1 The diagram shows a sector of a circle with centre O and radius 6 cm.

Angle $POQ = 0.6$ radians. Calculate the length of arc PQ and the area of sector POQ. (OCR)

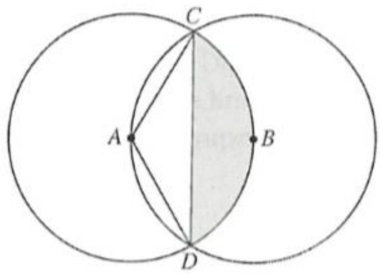


2 A sector OAB of a circle, of radius a and centre O, has $\angle AOB = \theta$ radians. Given that the area of the sector OAB is twice the square of the length of the arc AB, find θ. (OCR)

3 The diagram shows a sector of a circle, with centre O and radius r. The length of the arc is equal to half the perimeter of the sector. Find the area of the sector in terms of r. (OCR)

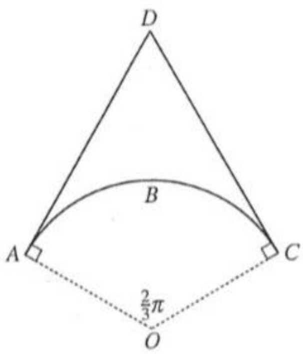


4 The diagram shows two circles, with centres A and B, intersecting at C and D in such a way that the centre of each lies on the circumference of the other. The radius of each circle is 1 unit. Write down the size of angle CAD and calculate the area of the shaded region (bounded by the arc CBD and the straight line CD). Hence show that the area of the region common to the interiors of the two circles is approximately 39% of the area of one circle. (OCR)



5 In the diagram, ABC is an arc of a circle with centre O and radius 5 cm. The lines AD and CD are tangents to the circle at A and C respectively. Angle $AOC = \frac{2}{3}\pi$ radians.

Calculate the area of the region enclosed by AD, DC and the arc ABC, giving your answer correct to 2 significant figures. (OCR)



6 Find, either to 2 decimal places or an exact multiple of π, all values of x in the interval $-\pi < x \leqslant \pi$ satisfying

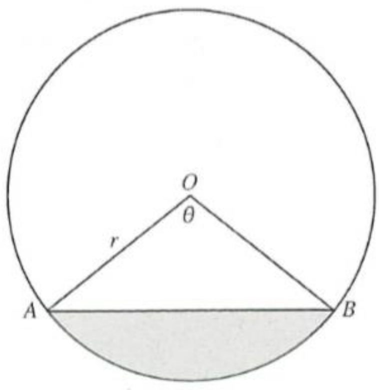
(a) $\sin x = -0.16$, (b) $\cos x\,(1 + \sin x) = 0$,

(c) $(1 - \tan x)\sin x = 0$, (d) $\sin 2x = 0.23$.

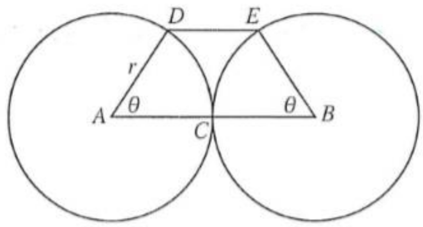
7 The diagram shows a circle with centre O and radius r, and a chord AB which subtends an angle θ radians at O. Express the area of the shaded segment bounded by the chord AB in terms of r and θ.

Given that the area of this segment is one-third of the area of triangle OAB, show that $3\theta - 4\sin\theta = 0$.

Find the positive value of θ satisfying $3\theta - 4\sin\theta = 0$ to within 0.1 radians, by tabulating values of $3\theta - 4\sin\theta$ and looking for a sign change, or otherwise. (OCR)



8 The diagram shows two circles, with centres A and B, which touch at C. The radius of each circle is r. The points D and E, one on each circle, are such that DE is parallel to the line ACB. Each of the angles DAC and EBC is θ radians, where $0 < \theta < \pi$. Express the length of DE in terms of r and θ.



The length of DE is equal to the length of each of the minor arcs CD and CE.

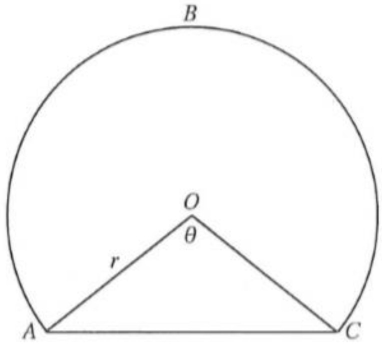
(a) Show that $\theta + 2\cos\theta - 2 = 0$.

(b) Sketch the graph of $y = \cos\theta$ for $0 < \theta < \frac{1}{2}\pi$. By drawing on your graph a suitable straight line, the equation of which must be stated, show that the equation $\theta + 2\cos\theta - 2 = 0$ has exactly one root in the interval $0 < \theta < \frac{1}{2}\pi$.

Verify by calculation that θ lies between 1.10 and 1.11. (OCR)

9 The diagram shows an arc ABC of a circle with centre O and radius r, and the chord AC. The length of the arc ABC is s, and angle $AOC = \theta$ rad. Express θ in terms of r and s, and deduce that the area of triangle AOC may be expressed as

$$\frac{1}{2}r^2\sin\left(2\pi - \frac{s}{r}\right).$$



Show, by a graphical argument based on a sketch of $y = \sin x$, or otherwise, that

$$\sin(2\pi - \alpha) = -\sin\alpha,$$

where α is any angle measured in radians.

Given that the area of triangle AOC is equal to one-fifth of the area of the major sector $OABC$, show that $\frac{s}{r} + 5\sin\left(\frac{s}{r}\right) = 0$.

10 Solve the equation $3\sin^2\theta + 4\cos\theta = 4$, giving all the roots, correct to 2 decimal places as appropriate, in the interval $0 \leqslant \theta \leqslant 2\pi$.

11 Solve the following equations giving any roots in terms of π in the interval $-2\pi \leqslant 0 \leqslant 2\pi$.

(a) $2\cos^2\theta + \sin^2\theta = 0$ (b) $2\cos^2\theta + \sin^2\theta = 1$ (c) $2\cos^2\theta + \sin^2\theta = 2$

1 3.6 cm, 10.8 cm^2

2 $\frac{1}{4}$

3 r^2

4 $\frac{2}{3}\pi$, $\frac{1}{3}\pi - \frac{1}{4}\sqrt{3}$

5 17 cm^2

6 (a) -2.98, -0.16 (b) $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$
(c) $-\frac{3}{4}\pi$, 0, $\frac{1}{4}\pi$, π
(d) -3.03, -1.69, 0.12, 1.45

7 $\frac{1}{2}r^2(\theta - \sin\theta)$, $1.2 < \theta < 1.3$

8 $DE = 2r - 2r\cos\theta$
(b) $y = 1 - \frac{1}{2}\theta$

9 $\theta = 2\pi - \frac{s}{r}$

10 0, 1.23, 5.05, 2π

11 (a) No roots
(b) $-\frac{3}{2}\pi$, $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$, $\frac{3}{2}\pi$
(c) -2π, $-\pi$, 0, π, 2π