Coordinates, points and lines (answers at end)

1 Show that the triangle formed by the points $(-2, 5)$, $(1, 3)$ and $(5, 9)$ is right-angled.

2 Find the coordinates of the point where the lines $2x + y = 3$ and $3x + 5y - 1 = 0$ meet.

3 A triangle is formed by the points $A(-1, 3)$, $B(5, 7)$ and $C(0, 8)$.

(a) Show that the angle ACB is a right angle.

(b) Find the coordinates of the point where the line through B parallel to AC cuts the x-axis.

4 $A(7, 2)$ and $C(1, 4)$ are two vertices of a square $ABCD$.

(a) Find in the form $ax + by = c$ the equation of the diagonal BD.

(b) Find the coordinates of B and of D.

5 A quadrilateral $ABCD$ is formed by the points $A(-3, 2)$, $B(4, 3)$, $C(9, -2)$ and $D(2, -3)$.

(a) Show that all four sides are equal in length.

(b) Show that $ABCD$ is not a square.

6 P is the point $(7, 5)$ and l_1 is the line with equation $3x + 4y = 16$.

(a) Find the equation of the line l_2 which passes through P and is perpendicular to l_1.

(b) Find the point of intersection of the lines l_1 and l_2.

(c) Find the perpendicular distance of P from the line l_1.

7 Prove that the triangle with vertices $(-2, 8)$, $(3, 20)$ and $(11, 8)$ is isosceles. Find its area.

8 The three straight lines $y = x$, $7y = 2x$ and $4x + y = 60$ form a triangle. Find the coordinates of its vertices.

9 Find the equation of the line through $(1, 3)$ which is parallel to $2x + 7y = 5$. Give your answer in the form $ax + by = c$.

10 Find the equation of the perpendicular bisector of the line joining $(2, -5)$ and $(-4, 3)$. Give your answer in the form $ax + by + c = 0$.

11 The points $A(1, 2)$, $B(3, 5)$, $C(6, 6)$ and D form a parallelogram. Find the coordinates of the mid-point of AC. Use your answer to find the coordinates of D.

12 The point P is the foot of the perpendicular from the point $A(0, 3)$ to the line $y = 3x$.

(a) Find the equation of the line AP.

(b) Find the coordinates of the point P.

(c) Find the perpendicular distance of A from the line $y = 3x$.

13 Points which lie on the same straight line are called collinear. Show that the points $(-1, 3)$, $(4, 7)$ and $(-11, -5)$ are collinear.

14 Find the equation of the straight line that passes through the points $(3, -1)$ and $(-2, 2)$, giving your answer in the form $ax + by + c = 0$. Hence find the coordinates of the point of intersection of the line and the x-axis. (OCR)

15 The coordinates of the points A and B are $(3, 2)$ and $(4, -5)$ respectively. Find the coordinates of the mid-point of AB, and the gradient of AB.

Hence find the equation of the perpendicular bisector of AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (OCR)

16 The curve $y = 1 + \dfrac{1}{2+x}$ crosses the x-axis at the point A and the y-axis at the point B.

(a) Calculate the coordinates of A and of B.

(b) Find the equation of the line AB.

(c) Calculate the coordinates of the point of intersection of the line AB and the line with equation $3y = 4x$. (OCR)

17 The straight line p passes through the point (10, 1) and is perpendicular to the line r with equation $2x + y = 1$. Find the equation of p.

Find also the coordinates of the point of intersection of p and r, and deduce the perpendicular distance from the point (10, 1) to the line r. (OCR)

18 Show by calculation that the points $P(0, 7)$, $Q(6, 5)$, $R(5, 2)$ and $S(-1, 4)$ are the vertices of a rectangle.

19 The line $3x - 4y = 8$ meets the y-axis at A. The point C has coordinates $(-2, 9)$. The line through C perpendicular to $3x - 4y = 8$ meets it at B. Calculate the area of the triangle ABC.

20 The points $A(-3, -4)$ and $C(5, 4)$ are the ends of the diagonal of a rhombus $ABCD$.

(a) Find the equation of the diagonal BD.

(b) Given that the side BC has gradient $\frac{5}{3}$, find the coordinates of B and hence of D.

21 Find the equations of the medians (see Exercise 1D Question 6) of the triangle with vertices (0, 2), (6, 0) and (4, 4). Show that the medians are concurrent (all pass through the same point).

22 The line l_1 passes through the points $A(4, 8)$ and $B(10, 26)$. Show that an equation for l_1 is $y = 3x - 4$.

The line l_1 intersects the line l_2, which has equation $y = 5x + 4$, at C. Find the coordinates of C.

23 The point A has coordinates (1, 7) and the point B has coordinates (3, 1). The mid-point of AB is P. Find the equation of the straight line which passes through P and which is perpendicular to the line $5y + x = 7$. Give your answer in the form $y = mx + c$. (OCR)

24 (a) The point A has coordinates (2, 3) and the line l_1 has equation $x + 4y = 31$. The line l_2 passes through A and is perpendicular to l_1. Find the equation of l_2 in the form $y = mx + c$.

(b) The lines l_1 and l_2 intersect the point M. Find the coordinates of M.

(c) The point A is a vertex of the square $ABCD$. The diagonals of the square intersect at M. Find the coordinates of C. (OCR)

25 The coordinates of A, B and C are $(-2, 3)$, (2, 5) and (4, 1) respectively.

(a) Find the gradients of the lines AB, BC and CA.

(b) Hence or otherwise show that the triangle ABC is a right-angled triangle. (OCR)

2 $(2, -1)$

3 (b) $(3.6, 0)$

4 (a) $3x - y = 9$ (b) $(3, 0)$ and $(5, 6)$

6 (a) $4x - 3y = 13$ (b) $(4, 1)$ (c) 5

7 Area $= 78$

8 $(0, 0)$, $(12, 12)$ and $(14, 4)$

9 $2x + 7y = 23$

10 $3x - 4y - 1 = 0$

11 $\left(3\frac{1}{2}, 4\right)$, $(4, 3)$

12 (a) $x + 3y = 9$ (b) $(0.9, 2.7)$ (c) $\sqrt{0.9}$

14 $3x + 5y - 4 = 0$, $\left(1\frac{1}{3}, 0\right)$

15 $\left(3\frac{1}{2}, -1\frac{1}{2}\right)$, -7, $x - 7y - 14 \doteq 0$

16 (a) $(-3, 0)$, $(0, 1.5)$ (b) $x - 2y + 3 = 0$
(c) $(1.8, 2.4)$

17 $x - 2y - 8 = 0$, $(2, -3)$, $\sqrt{80}$

19 25

20 (a) $x + y = 1$ (b) $(2, -1)$, $(0, 1)$

21 $y = 2$, $3x + 4y = 18$, $y = 3x - 8$

22 $(-4, -16)$

23 $y = 5x - 6$

24 (a) $y = 4x - 5$ (b) $(3, 7)$ (c) $(4, 11)$

25 (a) $\frac{1}{2}, -2, -\frac{1}{3}$
(b) As gradient $AB \times$ gradient $BC = -1$, AB is perpendicular to BC, so triangle ABC is right-angled.