Mixed practice 15

1 A curve has gradient $\frac{dy}{dx} = 3x - \sqrt{x}$ and passes through the point $(4, -1)$. Find the equation of the curve.

2 Find the indefinite integral $\int \frac{1 + x\sqrt{x}}{x^2}\, dx$.

3 Given that $f'(x) = (1-x)(\sqrt{x}+2)$, and that $f(1) = 3$, find an expression for $f(x)$.

4 **a** Find the exact value of $\int_{2}^{2\sqrt{3}} 3 - \frac{12}{x^2}\, dx$.

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

b The curve in the diagram has equation $y = 3 - \frac{12}{x^2}$. The curve crosses the x-axis at $x = 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 0$ and $y = 2$.
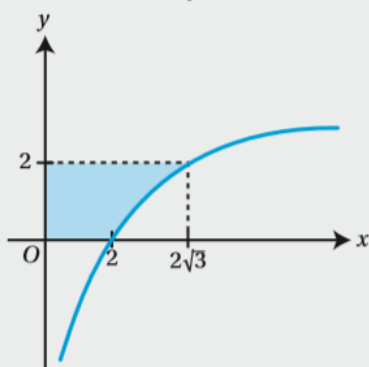
Find the area of the shaded region.

5 **i** Find $\int (x^2 - 2x + 5)\, dx$.

ii Hence find the equation of the curve for which $\frac{dy}{dx} = x^2 - 2x + 5$ and which passes through the point $(3, 11)$.

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6 Given that $f(x) = \int 4x + 5\, dx$, find $f'(x)$.

7 **a** Find $\int (x^2 + 4)(x - 6)\, dx$.

b The diagram shows the curve $y = 6x^{\frac{2}{3}}$ and part of the curve $y = \frac{8}{x^2} - 2$, which intersect at the point $(1, 6)$.

Use integration to find the area of the shaded region enclosed by the two curves and the x-axis.

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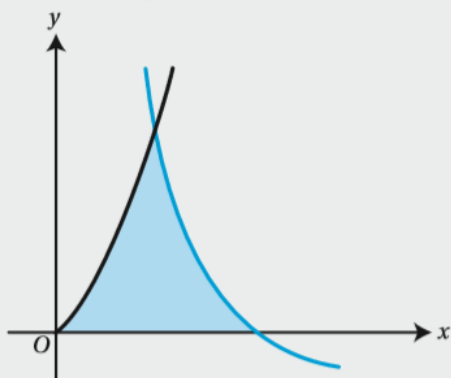
8 $f'(x) = \dfrac{4x^2 - 3\sqrt{x}}{x}$ and $f(1) = 2$.

Find $f(4)$.

9 $\displaystyle\int_1^a 2x - 3\,dx = 6,\ a > 0$

What is the value of a?

10 **a** Find the positive value of a for which $\displaystyle\int_0^a x^3 - x\,dx = 0$.

   b For this value of a, find the total area enclosed between the x-axis and the curve $y = x^3 - x$ for $0 \leqslant x \leqslant a$.

11 Find the area enclosed between the graph of $y = k^2 - x^2$ and the x-axis, giving your answer in terms of k.
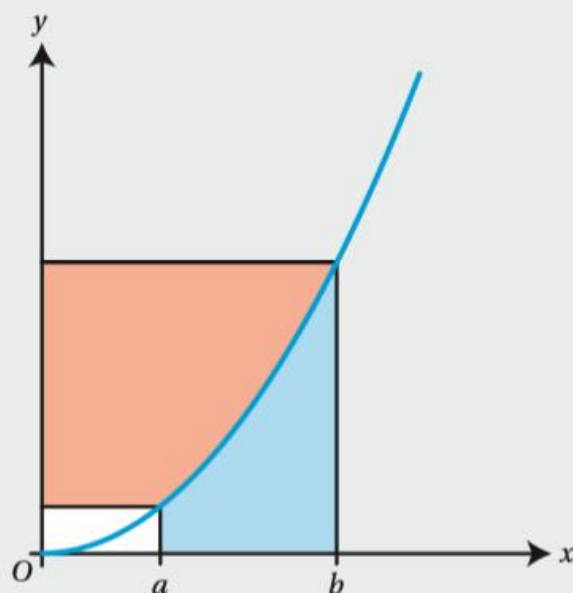
12 Let $f(x) = 2x^3 - 3x^2 - 3x + 2$.

   a i Show that $(x - 2)$ is a factor of $f(x)$ and hence factorise $f(x)$ completely.

   ii Sketch the graph of $y = f(x)$, labelling clearly the points where the curve crosses the coordinate axes.

   b Find the exact area enclosed by the x-axis and the graph of $y = f(x)$.

13 The diagram shows the graph of $y = x^n$ for $n > 1$.
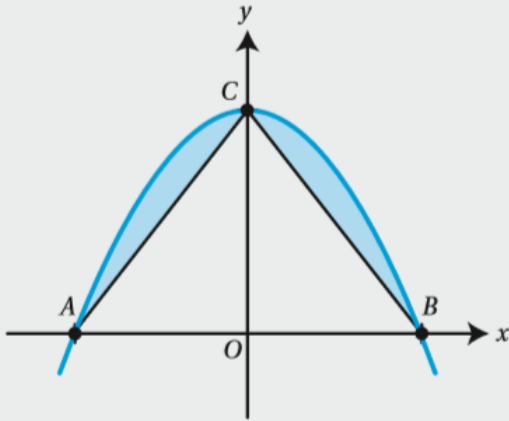

   **a i** Write down an expression for the area of the white rectangle.

   ii B is the area of the blue shaded region. Find an expression for B in terms of a, b and n.

   b The pink area is three times as large as the blue area. Find the value of n.

14 The diagram shows a parabola with equation $y = a^2 - x^2$. The parabola crosses the x-axis at points A and B, and the y-axis at point C.
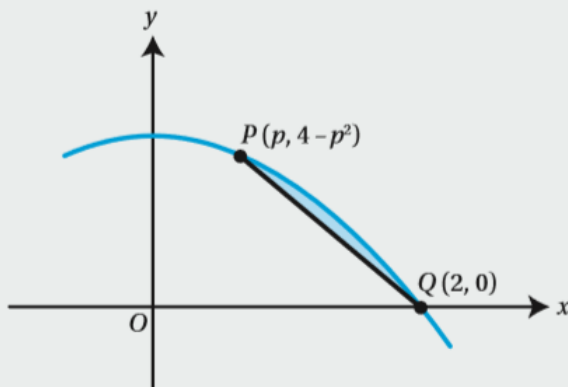

a i Write down the coordinates of A, B and C.

ii Find, in terms of a, the area of the shaded region.

b Point P lies on the parabola. The x-coordinate of P is p.
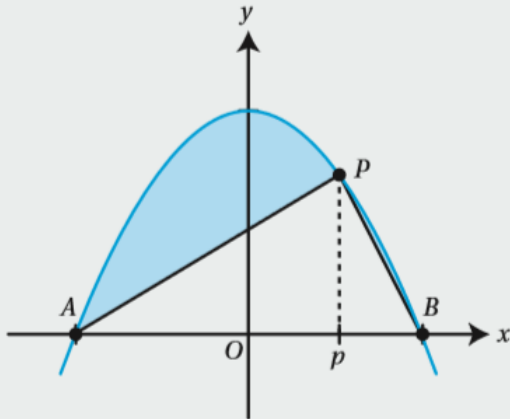
The value of p varies between the x-coordinates of A and B.

Find the minimum value of the shaded area.

15 A part of the curve with equation $y = 4 - x^2$ is shown in the diagram. Point P has coordinates $(p, 4 - p^2)$ and point Q has coordinates $(2, 0)$.


The shaded region is bounded by the curve and the chord PQ. Show that the area of the shaded region is $\frac{1}{6}(2-p)^3$.

16 The function $f(x)$ has a stationary point at $(3, 19)$ and $f''(x) = 6x + 6$.

  a Determine the nature of the stationary point at $(3, 19)$.

  b Find an expression for $f(x)$.

17 The diagram shows the graph of $y = 6\sqrt{x}$ and the tangent to the graph at the point $(9, 18)$. The tangent crosses the y-axis at the point B.
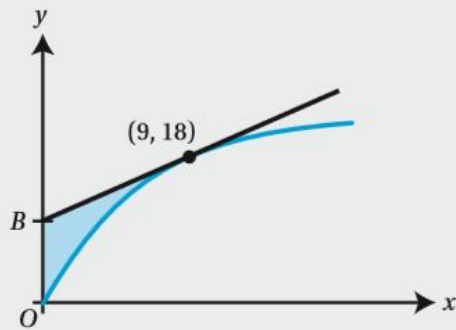
Find the area of the shaded region.

Mixed practice 15

1 $y = \dfrac{3}{2}x^2 - \dfrac{2}{3}x^{\frac{3}{2}} - \dfrac{59}{3}$

2 $-\dfrac{1}{x} + 2\sqrt{x} + c$

3 $\dfrac{2}{3}x^{\frac{3}{2}} - \dfrac{2}{5}x^{\frac{5}{2}} + 2x - x^2 + \dfrac{26}{15}$

4 a $-12 + 8\sqrt{3}$ **b** $12 - 4\sqrt{3}$

5 a $\dfrac{1}{3}x^3 - x^2 + 5x + c$ **b** $\dfrac{1}{3}x^3 - x^2 + 5x - 4$

6 $4x + 5$

7 a $\dfrac{1}{4}x^4 - 2x^3 + 2x^2 - 24x + c$
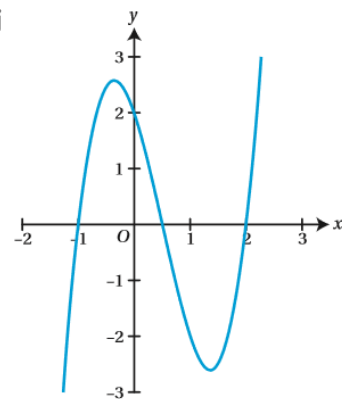
  b $\dfrac{28}{5}$

8 26

9 $a = 4$

10 a $a = \sqrt{2}$

  b $\dfrac{1}{2}$

11 $\dfrac{4k^3}{3}$

12 a i $(x - 2)(2x - 1)(x + 1)$

  ii

  b $\dfrac{81}{16}$

13 a i a^{n+1} **ii** $\dfrac{b^{n+1} - a^{n+1}}{n+1}$

  b $n = 3$

14 a i $A(-a, 0)$, $B(a, 0)$, $C(0, a^2)$

  ii $\dfrac{1}{3}a^3$

  b $\dfrac{1}{3}a^3$

15 Proof

16 a A (local) minimum

  b $x^3 + 3x^2 - 45x + 100$

17 $\dfrac{27}{2}$