Mixed practice 19

1 An object moves in a straight line so that its velocity, v m s^{-1}, is given by the equation $v=3t^2-8t$.

a Find the acceleration of the object after 2 seconds.

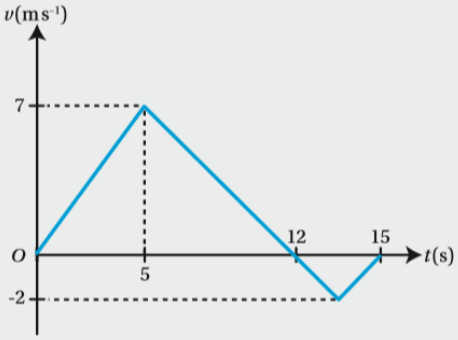
b Find the equation for the displacement from the initial position after t seconds.

2 A particle moves in a straight line with velocity $v=(3-t^{-2})$ m s^{-1} for $t \geqslant 1$.

a Find the distance travelled between $t=1$ and $t=5$.

b Hence find the average speed of the particle between $t=1$ and $t=5$.

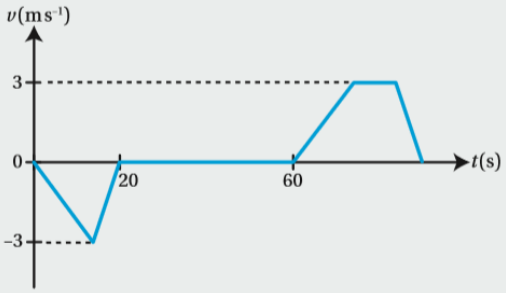
3 The motion of a particle moving in a straight line is represented on the following velocity–time graph:



a Find the acceleration of the particle between $t=0$ and $t=5$.

b State the times when the particle is instantaneously at rest.

c Find the average speed of the particle for the first 15 seconds.

4 

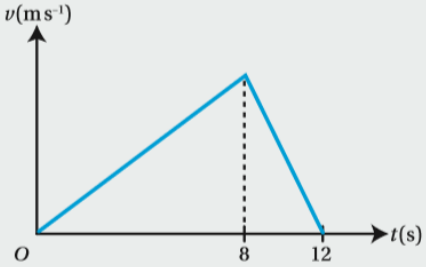
The diagram shows the (t, v) graph for a lorry delivering waste to a recycling centre. The graph consists of six straight line segments. The lorry reverses in a straight line from a stationary position on a weighbridge before coming to rest. It deposits its waste and then moves forwards in a straight line accelerating to a maximum speed of 3 m s^{-1}. It maintains this speed for 4 s and then decelerates, coming to rest at the weighbridge.

i Calculate the distance from the weighbridge to the point where the lorry deposits the waste.

ii Calculate the time which elapses between the lorry leaving the weighbridge and returning to it.

iii Given that the acceleration of the lorry when it is moving forwards is 0.4 m s^{-2}, calculate its final deceleration.

5 This velocity-time graph shows the motion of a particle moving in a straight line. The total distance travelled during the 12 seconds is 360 m.



Find the acceleration of the particle during the final 4 seconds.

6 A car travels along a straight road. Its velocity, in kilometres per hour, is given by $v=40+10t-0.5t^2$ (for $0 \leqslant t \leqslant 20$), where time is measured in seconds. It passes point A when $t=0$.

a Write an equation for the velocity in metres per second.

b Find the acceleration of the car in terms of t. Hence find the time when the car has maximum velocity.

c Find the distance of the car from A when $t=12$.

d The car is modelled as a particle. Explain whether this is a suitable modelling assumption in the following two questions.

i How long does the car take to overtake a stationary van of length 6.2 m?

ii How long does the car take to pass through a tunnel of length 380 m?

7 A bus travels in a straight line. When it passes a man its speed is 8.5 m s^{-1}. It decelerates uniformly until it comes to rest at the bus stop 44.2 m away.

As the bus passes the man, the man starts running at a constant velocity, V m s^{-1}. He arrives at the bus stop at the same time as the bus.

Find the value of V.

8 A model train travels along a straight track. At time t seconds after setting out from station A, the train has velocity v m s^{-1} and displacement x metres from A. It is given that for $0 \leqslant t \leqslant 7$

$$x=0.01t^4-0.16t^3+0.72t^2.$$

After leaving A the train comes to instantaneous rest at station B.

i Express v in terms of t. Verify that when $t=2$ the velocity of the train is 1.28 m s^{-1}.

ii Express the acceleration of the train in terms of t, and hence show that when the acceleration of the train is zero $t^2-8t+12=0$.

iii Calculate the minimum value of v.

iv Sketch the (t, v) graph for the train, and state the direction of motion of the train when it leaves B.

v Calculate the distance AB.

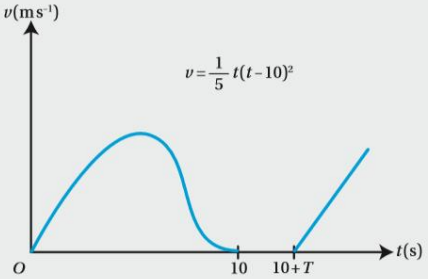
9 A particle moves with velocity $v\,\text{m s}^{-1}$, where:

$$v(t)=\begin{cases}0.16t^3-0.12t+10.6 & \text{for } 0\leqslant t\leqslant 5\\ 40-2t & \text{for } t>5\end{cases}$$

Find the two times when the particle is 200 m away from the starting point.

10 A car is travelling along a road that has a speed limit of 90 km h^{-1}. The speed of cars on the road is monitored via average speed check cameras, which calculate the average speed of a car by measuring how long it takes to travel a specified distance.

The car starts from rest next to one of the cameras. Its velocity in m s^{-1} is given by $v(t)=\frac{1}{5}t(t-10)^2$, and it comes to rest after 10 seconds. It stays stationary for T seconds and then starts moving again with a constant acceleration of 3.5 m s^{-2}. The velocity–time graph of the car's motion is shown in the diagram.



The second camera is positioned 300 m away from the first one.

a Find the time the car takes to reach the second camera after it has started from rest the second time.

b Show that the car's speed exceeded 90 km h^{-1} during both stages of motion.

c The cameras did not detect the car breaking the speed limit. Find the smallest possible value of T.

11 A particle is moving in a straight line so that its displacement from the starting point, x metres, is given by $x=0.8t^3-0.12t^4$.

Find the maximum speed of the particle during the first 6 seconds.

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1 **a** 4 m s^{-2} **b** $x=t^3-4t^2$

2 **a** 11.2 m **b** 2.8 m s^{-1}

3 **a** 1.4 m s^{-2} **b** 0, 12, 15 s
c 3 m s^{-1}

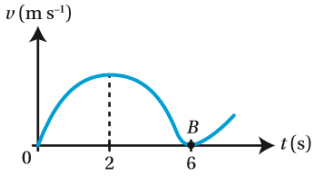
4 **a** 30 m **b** 76 s
c −0.667 m s^{-2}

5 −15 m s^{-2}

6 **a** $v=\frac{100}{9}+\frac{25}{9}t-\frac{5}{36}t^2$
b $a=\frac{25}{9}-\frac{5}{18}t;\ t=10$
c 253 m
d **i** No **ii** Yes

7 4.25

8 **a** $v=0.04t^3-0.48t^2+1.44t$
b $a=0.12t^2-0.96t+1.44$
c 0 m s^{-1}
d Forwards (away from A)



e 4.32 m

9 9.93 s, 30.1 s

10 **a** 8.73 s **b** Proof **c** 0 s

11 17.3 m s^{-1}