Mixed practice 14

1. $y = f(x)$ has a stationary point, P, at $x = 2$. What is the equation of the tangent at P?

   Choose from the following options:

   A $y = f'(2)$ **B** $x = 2$ **C** $y = f(2)$ **D** $x = -\frac{1}{2}$

2. Find the equation of the tangent to the curve $y = x^2 - \frac{3}{x}$ at the point where $x = 2$. Give your answer in the form $ax + by = c$ where a, b and c are integers.

3. Find the equation of the normal to the curve $y = (x-2)^3$ when $x = 2$.

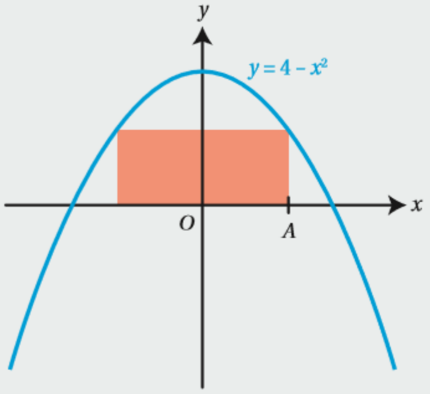
4. Find the x-coordinates of the stationary points on the graph of $y = \frac{x^3}{6} - x^2 + x$ and determine their nature.

5. A curve C has equation $y = \frac{1}{4}x^4 - 2x^3 + 4x^2 - 1$.

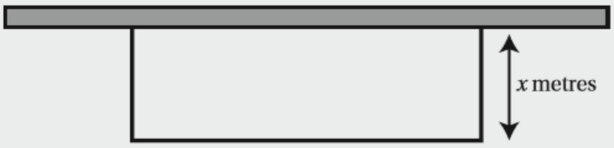
   a Find the coordinates of the stationary points on C.

   b Determine the nature of each stationary point.

6. A rectangle is drawn inside the region bounded by the curve $y = 4 - x^2$ and the x-axis, so that two of the vertices lie on the axis and the other two on the curve.



   Find the coordinates of vertex A so that the area of the rectangle is the maximum possible.

7. 

   The diagram shows a rectangular enclosure, with a wall forming one side. A rope, of length 20 metres, is used to form the remaining three sides. The width of the enclosure is x metres.

   i Show that the enclosed area, A m^2, is given by $A = 20x - 2x^2$.

   ii Use differentiation to find the maximum value of A.

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8. $y = \frac{1}{3}x^3 - ax^2 + 3ax + 1$ where $a \neq 0$ has one stationary point. What is the value of a?

9. The curve $y = ax^3 + bx^2 + 8x - 1$ has stationary points at $x = \frac{1}{3}$ and $x = 4$. Find a and b.

10. $f(x) = x^2 - 4x\sqrt{x} + 4x - 3$

    Show that the curve $y = f(x)$ has two stationary points and determine whether each is a maximum or minimum.

11 A function is defined by $f(x) = x^3 - 9x$ for $-2 \leqslant x \leqslant 5$.

a Find the coordinates of the stationary points on the curve $y = f(x)$.

b Find the minimum and maximum values of $f(x)$.

12 The tangent to the graph of $y = \frac{1}{x^2}$ at the point where $x = 3$ crosses the coordinate axes at points M and N. Find the exact area of the triangle MON.

13 A car tank is being filled with petrol such that the volume in the tank in litres (V) over time in minutes (t) is given by $V = 300(t^2 - t^3) + 4$ for $0 < t < 0.5$.

a How much petrol was initially in the tank?

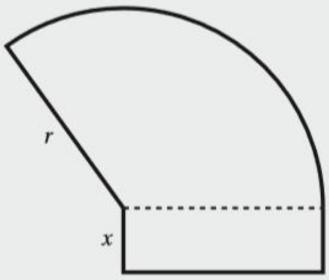
b After 30 seconds the tank was full. What is the capacity of the tank?

c At what time is petrol flowing in at the greatest rate?

14 A gardener is planting a lawn in the shape of a sector of a circle joined to a rectangle. The sector has radius r and angle $\frac{2\pi}{3}$ radians.

He needs the area, A, of the lawn to be 200 m^2.

A fence is to be built around the perimeter of the lawn.



a Show that the length of the fence, P, is given by $P = 2r + \frac{400}{r}$.

b Hence find the minimum length of fence required, justifying that this value is a minimum.

15 **i** Find the coordinates of the stationary point on the curve $y = 3x^2 - \frac{6}{x} - 2$.

ii Determine whether the stationary point is a maximum point or a minimum point.

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16 The line $y = 24(x - 1)$ is tangent to the curve $y = ax^3 + bx^2 + 4$ at $x = 2$.

a Use the fact that the tangent meets the curve to show that $2a + b = 5$.

b Use the fact that the tangent has the same gradient as the curve to find another relationship between a and b.

c Hence find the values of a and b.

d The line meets the curve again. Find the coordinates of the other point of intersection.

17 On the curve $y = x^3$ a tangent is drawn from the point (a, a^3) and a normal is drawn from the point $(-a, -a^3)$. The tangent and the normal meet on the y-axis. Find the value of a.

18 The curve $y = ax^2 + \frac{24}{x}$ has a stationary point at $y = 18$. Find a.

Mixed practice 14

1 C

2 $19x - 4y = 28$

3 $x = 2$

4 $x = 2 + \sqrt{2}$ local minimum; $x = 2 - \sqrt{2}$ local maximum

5 **a** $(0, -1)$ $(2, 3)$ $(4, -1)$
b $(0, -1)$ local minimum $(2, 3)$ local maximum $(4, -1)$ local minimum

6 $\left(\frac{2\sqrt{3}}{3}, 0\right)$

7 **a** Proof **b** $50\ \text{m}^2$

8 $a = 3$

9 $a = 2, b = -13$

10 $(1, -2)$ local maximum
$(4, -3)$ local minimum

11 **a** $\left(-\sqrt{3}, 6\sqrt{3}\right)$ and $\left(\sqrt{3}, -6\sqrt{3}\right)$
b Minimum: $-6\sqrt{3} \leqslant f(x) \leqslant 80$

12 $\frac{3}{4}$

13 **a** 4 litres **b** 41.5 litres
c 20 seconds

14 **a** Proof **b** $40\sqrt{2}$ m

15 **a** $(-1, 7)$ **b** Minimum

16 **a** Proof **b** $3a + b = 6$
c $a = 1, b = 3$ **d** $(-7, -192)$

17 $a = 3^{-\frac{1}{4}}$

18 $a = 1.5$