Investigating shapes of graphs (answers at the end)

**1** Use differentiation to find the coordinates of the stationary points on the curve

$$y = x + \frac{4}{x}$$

and determine whether each stationary point is a maximum point or a minimum point.

Find the set of values of $x$ for which $y$ increases as $x$ increases. (OCR)

**2** (a) Find the stationary points on the graph of $y = 12x + 3x^2 - 2x^3$ and sketch the graph.

(b) How does your sketch show that the equation $12x + 3x^2 - 2x^3 = 0$ has exactly three real roots?

(c) Use your graph to show that the equation $12x + 3x^2 - 2x^3 = -5$ also has exactly three real roots.

(d) For what range of values of $k$ does the equation $12x + 3x^2 - 2x^3 = k$ have
(i) exactly three real roots, (ii) only one real root?

**3** Find the coordinates of the stationary points on the graph of

$$y = x^3 - 12x - 12$$

and sketch the graph.

Find the set of values of $k$ for which the equation

$$x^3 - 12x - 12 = k$$

has more than one real root. (OCR)

**4** Find the coordinates of the stationary points on the graph of $y = x^3 + x^2$. Sketch the graph and hence write down the set of values of the constant $k$ for which the equation $x^3 + x^2 = k$ has three distinct real roots.

**5** Find the coordinates of the stationary points on the curve with equation $y = x(x-1)^2$. Sketch the curve.

Find the set of real values of $k$ such that the equation $x(x-1)^2 = k^2$ has exactly one real root. (OCR, adapted)

**6** It is given that $f(x) = 5x^3 - 15x^2$.

(a) Find $f'(x)$.

(b) Find $f''(x)$.

(c) Find the $x$-coordinates of the stationary points on the curve $y = 5x^3 - 15x^2$.

(d) Determine whether each stationary point is a maximum point or a minimum point. (OCR)

**7** (a) Find the coordinates of the stationary points on the curve $y = 2x^3 - 3x^2 - 12x - 7$.

(b) Determine whether each stationary point is a maximum point or a minimum point.

(c) It is given that $2x^3 - 3x^2 - 12x - 7$ can be written as $(x+1)^2(2x-7)$. Sketch the curve $y = (x+1)^2(2x-7)$.

(d) Write down the set of values of the constant $k$ for which the equation $2x^3 - 3x^2 - 12x - 7 = k$ has exactly one real solution. (OCR)

**8*** Find the coordinates of the stationary points on the graph of $y = 3x^4 - 4x^3 - 12x^2 + 10$, and sketch the graph. For what values of $k$ does the equation $3x^4 - 4x^3 - 12x^2 + 10 = k$ have

(a) exactly four roots, (b) exactly two roots?

**9*** Show that the stationary point on $y = ax^2 + bx + c$ has coordinates $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$.

Hence show that the condition for $ax^2 + bx + c = 0$ to have no roots is $b^2 - 4ac < 0$. (You should consider the cases $a > 0$ and $a < 0$ separately.)

**1** $(-2, -4)$ maximum, $(2, 4)$ minimum; $x \leqslant -2$ and $x \geqslant 2$

**2** (a) $(-1, -7)$, $(2, 20)$

(b) The graph crosses the $x$-axis 3 times.

(c) The graph has 3 intersections with the line $y = -5$.

(d) (i) $-7 < k < 20$ (ii) $k < -7$ and $k > 20$

**3** $(-2, 4)$, $(2, -28)$; $-28 \leqslant k \leqslant 4$

**4** $\left(-\frac{2}{3}, \frac{4}{27}\right)$, $(0, 0)$; $0 < k < \frac{4}{27}$

**5** $\left(\frac{1}{3}, \frac{4}{27}\right)$, $(1, 0)$; $k < -\frac{2}{9}\sqrt{3}$ and $k > \frac{2}{9}\sqrt{3}$

**6** (a) $15x^2 - 30x$

(b) $30x - 30$ (c) 0, 2

(d) $x = 0$ maximum, $x = 2$ minimum

**7** (a) $(-1, 0)$, $(2, -27)$

(b) $x = -1$ maximum, $x = 2$ minimum

(d) $k < -27$ and $k > 0$

**8** $(-1, 5)$, $(0, 10)$, $(2, -22)$

(a) $5 < k < 10$

(b) $-22 < k < 5$ and $k > 10$