Index notation (answers at the end)

1 Evaluate the following without using a calculator.

(a) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2}$ (b) $32^{-\frac{4}{5}}$ (c) $\left(4\frac{3}{2}\right)^{-\frac{1}{3}}$ (d) $\left(1\frac{7}{9}\right)^{1\frac{1}{2}}$

2 Simplify the following expressions.

(a) $\left(4p^{\frac{1}{4}}q^{-3}\right)^{\frac{1}{2}}$ (b) $\dfrac{(5b)^{-1}}{(8b^6)^{\frac{1}{3}}}$

(c) $(2x^6y^8)^{\frac{1}{4}} \times (8x^{-2})^{\frac{1}{4}}$ (d) $\left(m^{\frac{1}{3}}n^{\frac{1}{2}}\right)^2 \times \left(m^{\frac{1}{6}}n^{\frac{1}{3}}\right)^4 \times (mn)^{-2}$

3 Express $(9a^4)^{-\frac{1}{2}}$ as an algebraic fraction in simplified form. (OCR)

4 Express $\dfrac{1}{(\sqrt{a})^{\frac{4}{3}}}$ in the form a^n, stating the value of n. (OCR)

5 By letting $y = x^{\frac{1}{3}}$, or otherwise, find the values of x for which $x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} = 1$. (OCR)

6 Solve the equation $4^{2x} \times 8^{x-1} = 32$.

7 Given that, in standard form, $3^{236} \approx 4 \times 10^{112}$, and $3^{-376} \approx 4 \times 10^{-180}$, find approximations, also in standard form, for the following.

(a) 3^{376} (b) 3^{612} (c) $\left(\sqrt{3}\right)^{236}$ (d) $(3^{-376})^{\frac{5}{2}}$

8 The table below shows, for three of the planets in the solar system, details of their mean distance from the Sun and the time taken for one orbit round the Sun.

Planet	Mean radius of orbit r metres	Period of revolution T seconds
Mercury	5.8×10^{10}	7.6×10^6
Jupiter	7.8×10^{11}	3.7×10^8
Pluto	5.9×10^{12}	7.8×10^9

(a) Show that r^3T^{-2} has approximately the same value for each planet in the table.

(b) The Earth takes one year for one orbit of the Sun. Find the mean radius of the Earth's orbit around the Sun.

9 Simplify

(a) $2^{-\frac{3}{2}} + 2^{-\frac{1}{2}} + 2^{\frac{1}{2}} + 2^{\frac{3}{2}}$, giving your answer in the form $k\sqrt{2}$,

(b) $\left(\sqrt{3}\right)^{-3} + \left(\sqrt{3}\right)^{-2} + \left(\sqrt{3}\right)^{-1} + \left(\sqrt{3}\right)^{0} + \left(\sqrt{3}\right)^{1} + \left(\sqrt{3}\right)^{2} + \left(\sqrt{3}\right)^{3}$, giving your answer in the form $a + b\sqrt{3}$.

10 Express each of the following in the form 2^n.

(a) $2^{70} + 2^{70}$ (b) $2^{-400} + 2^{-400}$ (c) $2^{\frac{1}{3}} + 2^{\frac{1}{3}} + 2^{\frac{1}{3}} + 2^{\frac{1}{3}}$

(d) $2^{100} - 2^{99}$ (e) $8^{0.1} + 8^{0.1} + 8^{0.1} + 8^{0.1} + 8^{0.1} + 8^{0.1} + 8^{0.1} + 8^{0.1}$

11 Solve the equation $\dfrac{125^{3x}}{5^{x+4}} = \dfrac{25^{x-2}}{3125}$.

12 The formulae for the volume V and the surface area S of a cube are $V = x^3$ and $S = 6x^2$, where x is the length of an edge. Find expressions for

(a) S in terms of V, (b) V in terms of S,

giving each answer in the form (S or V) $= 2^m \times 3^n \times (V$ or $S)^p$.

13 Einstein's law $E = mc^2$ gives the energy of the radiation created by the destruction of a particle of mass m, where c is the velocity of light. The units for m, c and E are respectively kilograms, metres per second and joules. Given that the speed of light is 3.0×10^8 metres per second, find the energy created by a neutron of mass 1.7×10^{-27} kilograms.

1 (a) 6 (b) $\frac{1}{16}$ (c) $\frac{1}{2}$ (d) $2\frac{10}{27}$

2 (a) $2p^{\frac{1}{8}}q^{-\frac{3}{2}}$ (b) $\frac{1}{10}b^{-3}$
(c) $2xy^2$ (d) $m^{-\frac{2}{3}}n^{\frac{1}{3}}$

3 $\dfrac{1}{3a^2}$

4 $a^{-\frac{2}{3}}$

5 -1 and 8

6 $x = \frac{8}{7}$

7 (a) 2.5×10^{179} (b) 1×10^{292}
(c) 2×10^{56} (d) 3×10^{-449}

8 (b) 1.5×10^{11} m (to 2 significant figures)

9 (a) $\frac{15}{4}\sqrt{2}$ (b) $\frac{13}{3} + \frac{40}{9}\sqrt{3}$

10 (a) 2^{71} (b) 2^{-399} (c) $2^{\frac{7}{3}}$
(d) 2^{99} (e) $2^{3.3}$

11 $x = -\frac{5}{6}$

12 (a) $S = 2^1 \times 3^1 \times V^{\frac{2}{3}}$
(b) $V = 2^{-\frac{3}{2}} \times 3^{-\frac{3}{2}} \times S^{\frac{3}{2}}$

13 1.5×10^{-10} joules