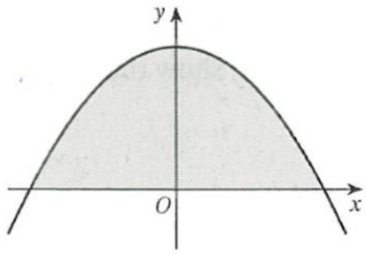
Integration (answers at the end)

1 A curve passes through (2, 3) and is such that $\frac{dy}{dx} = \frac{1}{2}x^2 - \frac{1}{3}x + 1$. Find y in terms of x.

2 The diagram shows the graph of $y = 12 - 3x^2$. Determine the x-coordinate of each of the points where the curve crosses the x-axis. Find by integration the area of the region (shaded in the diagram) between the curve and the x-axis. (OCR)



3 Find $\int 6\sqrt{x}\, dx$ and hence evaluate $\int_1^4 6\sqrt{x}\, dx$. (OCR)

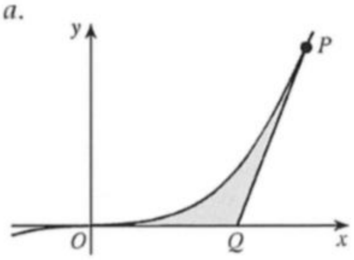
4 (a) Find $\int \left(\frac{1}{x^3} + x^3\right) dx$. (b) Evaluate $\int_0^8 \frac{1}{\sqrt[3]{x}}\, dx$. (OCR)

5 Find the area of the region enclosed between the curve $y = 12x^2 + 30x$ and the x-axis.

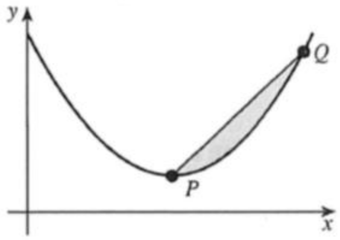
6 Find the value of the improper integral $\int_0^4 \frac{(x+2)^2}{\sqrt{x}}\, dx$.

7 Given that $\int_{-a}^{a} 15x^2\, dx = 3430$, find the value of the constant a.

8 The diagram shows the curve $y = x^3$. The point P has coordinates (3, 27) and PQ is the tangent to the curve at P. Find the area of the region enclosed between the curve, PQ and the x-axis.



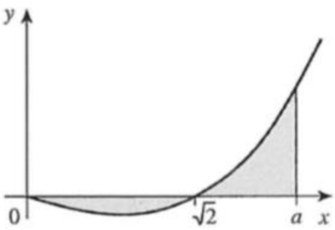
9 The diagram shows the curve $y = (x-2)^2 + 1$ with minimum point P. The point Q on the curve is such that the gradient of PQ is 2. Find the area of the region, shaded in the diagram, between PQ and the curve.



10 Evaluate $\int_0^2 x(x-1)(x-2)\, dx$ and explain your answer with reference to the graph of $y = x(x-1)(x-2)$.

11 (a) Find $\int x(x^2 - 2)\, dx$.

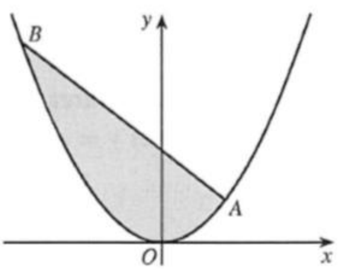
(b) The diagram shows the graph of $y = x(x^2 - 2)$ for $x \geqslant 0$. The value of a is such that the two shaded regions have equal areas. Find the value of a. (OCR)



12 The diagram shows a sketch of the graph of $y = x^2$ and the normal to the curve at the point $A(1, 1)$.

(a) Use differentiation to find the equation of the normal at A. Verify that the point B where the normal cuts the curve again has coordinates $\left(-\frac{3}{2}, \frac{9}{4}\right)$.

(b) The region which is bounded by the curve and the normal is shaded in the diagram. Calculate its area, giving your answer as an exact fraction. (OCR)



13 Given that $\int_1^p (8x^3 + 6x)\, dx = 39$, find two possible values of p. Use a graph to explain why there are two values.

14 Show that the area enclosed between the curves $y = 9 - x^2$ and $y = x^2 - 7$ is $\frac{128\sqrt{2}}{3}$.

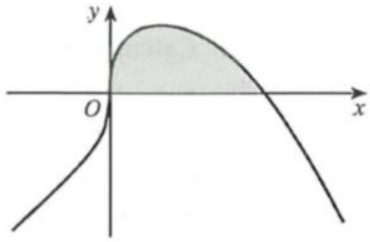
15 Given that $f(x)$ and $g(x)$ are two functions such that $\int_0^4 f(x)\,dx = 17$ and $\int_0^4 g(x)\,dx = 11$, find, where possible, the value of each of the following.

(a) $\int_0^4 (f(x) - g(x))\,dx$ (b) $\int_0^4 (2f(x) + 3g(x))\,dx$ (c) $\int_0^2 f(x)\,dx$

(d) $\int_0^4 (f(x) + 2x + 3)\,dx$ (e) $\int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx$ (f) $\int_4^0 g(x)\,dx$

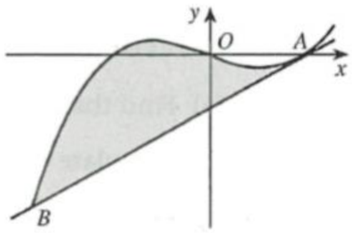
(g)* $\int_1^5 f(x-1)\,dx$ (h)* $\int_{-4}^0 g(-t)\,dt$

16 The diagram shows the graph of $y = \sqrt[3]{x} - x^2$. Show by integration that the area of the region (shaded in the diagram) between the curve and the x-axis is $\frac{5}{12}$. (OCR)



17 The diagram shows a sketch of the graph of the curve $y = x^3 - x$ together with the tangent to the curve at the point $A(1, 0)$.

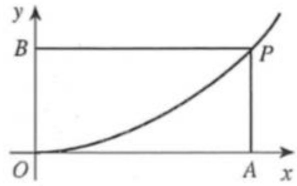
(a) Use differentiation to find the equation of the tangent to the curve at A, and verify that the point B where the tangent cuts the curve again has coordinates $(-2, -6)$.



(b) Use integration to find the area of the region bounded by the curve and the tangent (shaded in the diagram), giving your answer as a fraction in its lowest terms. (OCR)

18* The diagram shows part of the curve $y = x^n$, where $n > 1$.

The point P on the curve has x-coordinate a. Show that the curve divides the rectangle $OAPB$ into two regions whose areas are in the ratio $n : 1$.



19* Find the stationary points on the graph of $y = x^4 - 8x^2$. Use your answers to make a sketch of the graph. Show that the graphs of $y = x^4 - 8x^2$ and $y = x^2$ enclose two finite regions. Find the area of one of them.

20* Using the same axes, make sketches of the graphs of $y = x^3$ and $y = (x+1)^3 - 1$. Then sketch on a larger scale the finite area enclosed between them.

Given that $(x+1)^3 = x^3 + 3x^2 + 3x + 1$, find the area of the region.

21* A function $f(x)$ is defined by $f(x) = \frac{6}{x^4} - \frac{2}{x^3}$ for $x > 0$.

(a) Find the values of $\int_2^3 f(x)\,dx$ and $\int_2^\infty f(x)\,dx$.

(b) Find the coordinates of (i) the point where the graph of $y = f(x)$ crosses the x-axis, (ii) the minimum point on the graph.

Use your answers to draw a sketch of the graph, and hence explain your answers to part (a).

1 $y = \frac{1}{6}x^3 - \frac{1}{6}x^2 + x + \frac{1}{3}$

2 ± 2; 32

3 $4x\sqrt{x} + k$; 28

4 (a) $-\frac{1}{2}x^{-2} + \frac{1}{4}x^4 + k$ (b) 6

5 $31\frac{1}{4}$

6 $50\frac{2}{15}$

7 7

8 $6\frac{3}{4}$

9 $1\frac{1}{3}$

10 0; integrand is positive for $0 < x < 1$, negative for $1 < x < 2$.

11 (a) $\frac{1}{4}x^4 - x^2 + k$ (b) 2

12 (a) $y = -\frac{1}{2}x + \frac{3}{2}$ (b) $2\frac{29}{48}$

13 ± 2

15 (a) 6 (b) 67 (c) — (d) 45
(e) 17 (f) −11 (g) 17 (h) 11

17 (a) $y = 2x - 2$ (b) $6\frac{3}{4}$

19 (0, 0), ($\pm$2, −16); 32.4

20 $\frac{1}{2}$

21 (a) $\frac{1}{27}$, 0
(b) (i) (3,0) (ii) $\left(4, -\frac{1}{128}\right)$

The minimum at $x = 4$ shows that the function takes both positive and negative values when $x > 2$.