Graphs of nth power functions (answers at the end)

1 Find the equation of the normal to the curve with equation $y = \sqrt{x}$ at the point $(1, 1)$. Calculate the coordinates of the point at which this normal meets the graph of $y = -\sqrt{x}$.

2 For the curve $y = \dfrac{4}{x^2}$, find

(a) the equation of the tangent at $(-2\sqrt{2}, \frac{1}{2})$,

(b) the equation of the normal at $(\sqrt{2}, 2)$.

Show that the lines in parts (a) and (b) are the same. Illustrate this with a sketch.

3 On the curve $y = \dfrac{1}{x}$, P is the point at which $x = p$. Find the following, in terms of p.

(a) The y-coordinate of P.

(b) The value of $\dfrac{dy}{dx}$ at P.

(c) The equation of the tangent at P.

(d) The coordinates of the points Q and R where the tangent meets the x- and y-axes.

(e) The area of the triangle OQR.

4 Find the equations of the tangent and the normal to the graph with equation $y = \dfrac{1}{x} - \dfrac{4}{x^2}$ at the points where $x = 1$ and $x = 8$.

5 (a) Show that $\sqrt{2x} = \sqrt{2}\sqrt{x}$. Hence differentiate $\sqrt{2x}$, giving your answer in surd form.

(b) Differentiate $\sqrt{4x^3}$ and $\sqrt[3]{4x^4}$, giving your answers in surd form.

6 Differentiate the following, giving your answers in index form.

(a) $\dfrac{\sqrt{x} - 1}{x^2}$ (b) $\dfrac{x\sqrt{x} - 1}{x\sqrt{x}}$ (c) $\left(\dfrac{\sqrt{x} - 1}{x^2}\right)^2$ (d) $\left(\dfrac{x\sqrt{x} - 1}{x\sqrt{x}}\right)^2$

7 The tangents at $x = \frac{1}{4}$ to $y = \sqrt{x}$ and $y = \dfrac{1}{\sqrt{x}}$ meet at P. Find the coordinates of P.

8 The normals at $x = 2$ to $y = \dfrac{1}{x^2}$ and $y = \dfrac{1}{x^3}$ meet at Q. Find the coordinates of Q.

9 (a) Draw a sketch to show the graphs of $y = \dfrac{1}{x^2}$ and $y = \sqrt{x}$ and their point of intersection at the point $P(1, 1)$. Find the gradient of each curve at P, and show that the tangent at P to each curve is the normal to the other curve.

(b) The graphs of $y = x^m$ and $y = x^n$ intersect at the point $P(1, 1)$. Find the connection between m and n if the tangent to each curve at P is the normal to the other curve.

1 $y + 2x = 3$; $(\frac{9}{4}, -\frac{3}{2})$

2 (a),(b) $2\sqrt{2}y = x + 3\sqrt{2}$

3 (a) $\dfrac{1}{p}$ (b) $-\dfrac{1}{p^2}$

(c) $p^2y + x = 2p$ (d) $(2p, 0), \left(0, \dfrac{2}{p}\right)$

(e) 2

4 $y = 7x - 10$, $7y + x = -20$; $y = \frac{1}{16}$, $x = 8$

5 (a) $\dfrac{1}{\sqrt{2x}}$ (b) $3\sqrt{x}$, $\frac{4}{3}\sqrt[3]{4x}$

6 (a) $-\frac{3}{2}x^{-\frac{5}{2}} + 2x^{-3}$ (b) $\frac{3}{2}x^{-\frac{5}{2}}$

(c) $-3x^{-4} + 7x^{-\frac{9}{2}} - 4x^{-5}$ (d) $3x^{-\frac{5}{2}} - 3x^{-4}$

7 $(\frac{11}{20}, \frac{4}{5})$

8 $(\frac{67}{32}, \frac{5}{8})$

9 (b) $mn = -1$