## Mixed practice 12

1 Points A and B have position vectors $\mathbf{a} = 3\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = 2\mathbf{j}$. Find the exact distance between A and B.

2 **a** Given the points $P(-5, 2)$ and $Q(1, -3)$, write vector $\overrightarrow{PQ}$ in the form $a\mathbf{i} + b\mathbf{j}$.

b Point R is such that $\overrightarrow{RQ} = \mathbf{i} - 4\mathbf{j}$. Find the coordinates of R.

3 Points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 12 \\ -7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

M is the midpoint of AB.

a Find the position vector of M.

b Find the exact distance BM.

4 Points A, B and C have position vectors $\mathbf{a} = 3\mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$ and $\mathbf{c} = 4\mathbf{i} + \mathbf{j}$. Point D is such that $ABCD$ is a parallelogram. Find the position vector of D.

5 The diagram shows points P, Q and R such that $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PR} = 3\mathbf{b}$.

Points M and N are on PQ and PR such that $PM = \frac{1}{3}PQ$ and $\overrightarrow{PN} = \mathbf{b}$.
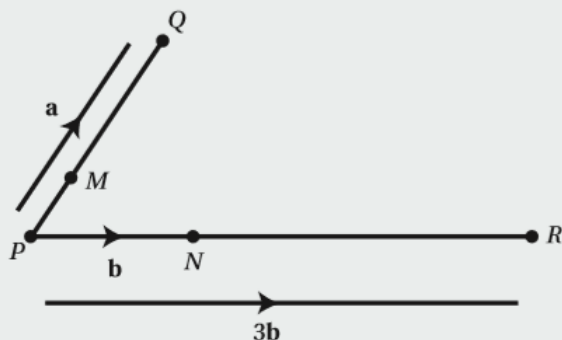
Express $\overrightarrow{MN}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and hence prove that MN is parallel to QR.

6 OAB is a triangle with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AB and G is a point on OM such that $OG : GM = 2 : 1$. N is the midpoint of OA. Use vectors to prove that the points B, G and N are collinear.

7 Find the magnitude of the vector $(3 \sin \theta)\mathbf{i} + (5 \cos \theta)\mathbf{j}$ in terms of θ.

8 Points M and N have coordinates $M(-6, 1)$ and $N(3, 5)$. Find a unit vector parallel to $\overrightarrow{MN}$.

9 Points P and Q have coordinates $(1, -8)$ and $(10, -2)$. N is a point on PQ such that $PN : NQ = 1 : 2$.

a Find the coordinates of N.

b Calculate the magnitudes of $\overrightarrow{OP}$, $\overrightarrow{ON}$ and $\overrightarrow{PN}$. Hence show that ONP is a right angle.

10 Points A, B, C and D have position vectors **a**, **b**, **c** and **d**. M, N, P and Q are midpoints of AB, BC, CD and DA.

   a Express vectors $\overrightarrow{MN}$ and $\overrightarrow{PQ}$ in terms of **a**, **b**, **c** and **d**.

   b What type of quadrilateral is $MNPQ$?
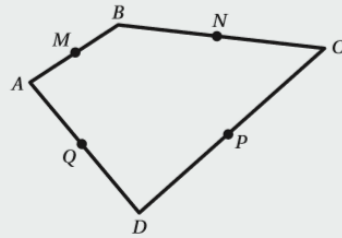
11 Points A and B have position vectors **a** and **b**. O is the origin and point P is such that $OAPB$ is a parallelogram.

   a Write down the position vector of P in terms of **a** and **b**.

   b Find the position vector of M, the midpoint of AP.

      Point Q lies on OP. Let $\overrightarrow{OQ}=t\overrightarrow{OP}$.

   c Express $\overrightarrow{BQ}$ in terms of t, **a** and **b**. Hence find the value of t for which BQM is a straight line.
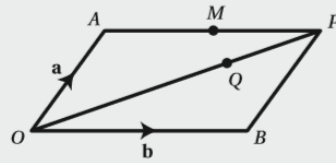
12 Points P and Q have position vectors $\mathbf{p}=\begin{pmatrix}5\\2\end{pmatrix}$ and $\mathbf{q}=\begin{pmatrix}2\\3\end{pmatrix}$.

   Point H lies on PQ and $\overrightarrow{PH}=t\overrightarrow{PQ}$, with $0<t<1$.

   Let O be the origin.

   a Express the vector $\overrightarrow{OH}$ and its length in terms of t.

   b Hence find the minimum possible distance of H from the origin, giving your answer in exact form.

13 Points A and B have position vectors **a** and **b**. O is the origin and point P is such that $OAPB$ is a parallelogram.
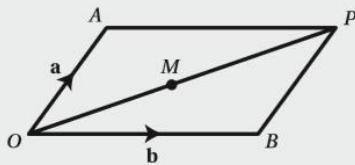
M is the midpoint of OP.

   a Show that M lies on AB and determine the ratio $AM:MB$.

   b What conclusion can you make about the diagonals of a parallelogram?

Mixed practice 12

1 $3\sqrt{2}$

2 a $6\mathbf{i}-5\mathbf{j}$ **b** $(0,1)$

3 a $\begin{pmatrix}4.5\\-1\end{pmatrix}$ **b** $\dfrac{3}{2}\sqrt{41}$

4 $6\mathbf{i}-2\mathbf{j}$

5 $\overrightarrow{MN}=\mathbf{b}-\dfrac{1}{3}\mathbf{a}$

6 Proof

7 $\sqrt{9+16\cos^2\theta}$

8 $\pm\left(\dfrac{9}{\sqrt{97}}\mathbf{i}+\dfrac{4}{\sqrt{97}}\mathbf{j}\right)$

9 a $(4,-6)$

   b $\overrightarrow{OP}=\sqrt{65},\overrightarrow{ON}=2\sqrt{13},\overrightarrow{PN}=\sqrt{13}$

10 a $\overrightarrow{MN}=\dfrac{1}{2}(\mathbf{c}-\mathbf{a}),\overrightarrow{PQ}=\dfrac{1}{2}(\mathbf{a}-\mathbf{c})$

   b Parallelogram

11 a $\mathbf{a}+\mathbf{b}$ **b** $\mathbf{a}+\dfrac{1}{2}\mathbf{b}$

   c $t\mathbf{a}+(t-1)\mathbf{b};t=\dfrac{2}{3}$

12 a $\begin{pmatrix}5-3t\\2+t\end{pmatrix};\sqrt{10t^2-26t+29}$ **b** $\dfrac{11\sqrt{10}}{10}$

13 a $1:1$

   b Diagonals bisect each other