Factors and remainders (answers at the end)

1 It is given that

$$(x + a)(x^2 + bx + 2) \equiv x^3 - 2x^2 - x - 6$$

where a and b are constants. Find the value of a and the value of b. (OCR)

2 Find the coordinates of the points where the graph of $y = 2x^3 + 3x^2 - 4x + 1$ cuts the x-axis.

3 Show that $(x - 1)$ is a factor of $6x^3 + 11x^2 - 5x - 12$, and find the other two linear factors of this expression. (OCR)

4 The cubic polynomial $x^3 + ax^2 + bx - 8$, where a and b are constants, has factors $(x + 1)$ and $(x + 2)$. Find the values of a and b. (OCR)

5 Find the value of a for which $(x - 2)$ is a factor of $3x^3 + ax^2 + x - 2$. Show that, for this value of a, the cubic equation $3x^3 + ax^2 + x - 2 = 0$ has only one real root. (OCR)

6 Solve the equation $4x^3 + 8x^2 + x - 3 = 0$ given that one of the roots is an integer. (OCR)

7 The cubic polynomial $x^3 - 2x^2 - 2x + 4$ has a factor $(x - a)$, where a is an integer.

(a) Use the factor theorem to find the value of a.

(b) Hence find exactly all three roots of the cubic equation $x^3 - 2x^2 - 2x + 4 = 0$. (OCR)

8 The cubic polynomial $x^3 - 2x^2 - x - 6$ is denoted by $f(x)$. Show that $(x - 3)$ is a factor of $f(x)$. Factorise $f(x)$. Hence find the number of real roots of the equation $f(x) = 0$, justifying your answer.

Hence write down the number of points of intersection of the graphs with equations

$$y = x^2 - 2x - 1 \quad \text{and} \quad y = \frac{6}{x},$$

justifying your answer. (OCR)

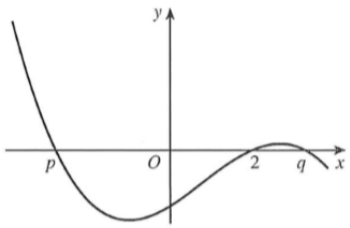
9 Given that $(2x + 1)$ is a factor of $2x^3 + ax^2 + 16x + 6$, show that $a = 9$.

Find the real quadratic factor of $2x^3 + 9x^2 + 16x + 6$. By completing the square, or otherwise, show that this quadratic factor is positive for all real values of x. (OCR)

10 Find the coordinates of the turning points on the curve with equation $y = 2x^4 - 7x^2 - 6x$.

11 The diagram shows the curve

$$y = -x^3 + 2x^2 + ax - 10.$$



The curve crosses the x-axis at $x = p$, $x = 2$ and $x = q$.

(a) Show that $a = 5$.

(b) Find the exact values of p and q. (OCR)

12 The polynomial $x^3 + 3x^2 + ax + b$ leaves a remainder of 3 when it is divided by $x + 1$ and a remainder of 15 when it is divided by $x - 2$. Find the remainder when it is divided by $x + 3$.

13 Find the maximum and minimum points on the curve with equation $y = 3x^2 + 14x + \frac{8}{x}$.

14 Let $p(x) = 4x^3 + 12x^2 + 5x - 6$.

(a) Calculate $p(2)$ and $p(-2)$, and state what you can deduce from your answers.

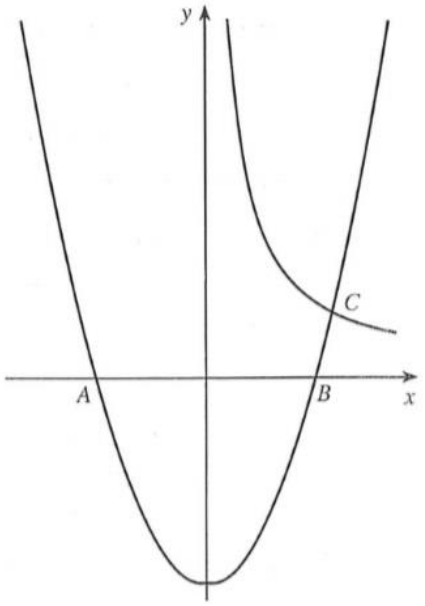
(b) Solve the equation $4x^3 + 12x^2 + 5x - 6 = 0$.

15 On the curve with equation $y = x^2(x - 4)$ the point P has coordinates $(1, -3)$.

(a) Find the equation of the tangent to the curve at P and the coordinates of the point where the tangent meets the curve again.

(b) Find the equation of the normal to the curve at P and the coordinates of the points where the normal meets the curve again.

16 The diagram shows the graph of $y = x^2 - 3$ and the part of the graph of $y = \frac{2}{x}$ for $x > 0$.



The two graphs intersect at C, and A and B are the points of intersection of $y = x^2 - 3$ with the x-axis. Write down the exact coordinates of A and B.

Show that the x-coordinate of C is given by the equation $x^3 - 3x - 2 = 0$.

Factorise $x^3 - 3x - 2$ completely.

Hence

(a) write down the x-coordinate of C,

(b) describe briefly the geometrical relationship between the graph of $y = x^2 - 3$ and the part of the graph of $y = \frac{2}{x}$ for which $x < 0$. (OCR)

1 $a = -3, b = 1$

2 $(\frac{1}{2}, 0), (\sqrt{2} - 1, 0), (-\sqrt{2} - 1, 0)$

3 $3x + 4, 2x + 3$

4 $a = -1, b = -10$

5 -6

6 $-1, \frac{1}{2}, -\frac{3}{2}$

7 (a) 2
(b) $2, \sqrt{2}, -\sqrt{2}$

8 $(x - 3)(x^2 + x + 2)$; one root only as the discriminant of the quadratic is negative; one point only, as the equation for the intersections is the given cubic.

9 $x^2 + 4x + 6 \equiv (x + 2)^2 + 2$

10 $(-1, 1), (-\frac{1}{2}, 1\frac{3}{8}), (1\frac{1}{2}, -14\frac{5}{8})$

11 (b) $p = -\sqrt{5}, q = \sqrt{5}$

12 5

13 $(-2, -20)$ minimum, $(-1, -19)$ maximum, $(\frac{2}{3}, 22\frac{2}{3})$ minimum

14 (a) 84, 0; the remainder when p(x) is divided by $x - 2$ is 84; $x + 2$ is a factor of p(x).
(b) $-2, -1\frac{1}{2}, \frac{1}{2}$

15 (a) $y + 5x = 2$, $(2, -8)$
(b) $5y - x = -16$, $(\frac{3}{2} \pm \frac{1}{10}\sqrt{545}, -\frac{29}{10} \pm \frac{1}{50}\sqrt{545})$

16 $A(-\sqrt{3}, 0)$, $B(\sqrt{3}, 0)$; $(x - 2)(x + 1)^2$
(a) 2
(b) They touch at $(-1, -2)$.