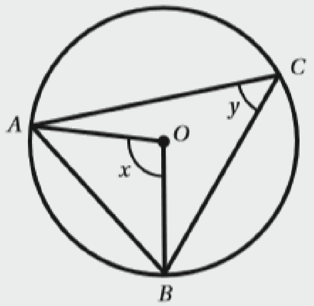
Mixed practice 1

1. Prove that the product of any two odd numbers is always odd.
2. Prove that if n is even then n^2 is divisible by 4.
3. Prove that if $\frac{a}{b} = \frac{c}{d}$ it does not follow that $a = c$ and $b = d$.
4. Prove the following statement or disprove it with a counter example:

   'The sum of two numbers is always larger than their difference.'
5. Prove that the product of two rational numbers is always rational.
6. Prove that the sum of the interior angles in an n-sided shape is $(180n - 360)°$.
7. Given that $x^3 + y^3 \equiv (x + y)(ax^2 + bxy + cy^2)$ find the values of a, b and c.
8. Prove the following statement:

   n is odd $\Rightarrow n^2 + 4n + 3$ is a multiple of 4
9. Prove that the angle from a chord to the centre of a circle is twice the angle to a point on the circumference in the major sector.



10. Prove that all cube numbers are either multiples of 9 or within one of a multiple of 9.
11. Prove the following statements, or disprove them with a counter example:
    - a ab is an integer $\Leftrightarrow$ a is an integer and b is an integer
    - b a is irrational and b is irrational $\Leftrightarrow$ ab is irrational.
12. Prove that the product of any three consecutive positive integers is a multiple of 6.
13. Prove that the difference between the squares of any two odd numbers is a multiple of 8.
14. a Prove that $n^2 - 79n + 1601$ is not always prime when n is a positive whole number.

    b Prove that $n^2 - 1$ is never prime when n is a whole number greater than 2.
15. $x = a^2 - b^2$ where a and b are both whole numbers. Prove that x is either odd or a multiple of 4.

Mixed practice 1

1 Use $a = 2m + 1$, $b = 2n + 1$

2 Use $n = 2m$

3 Use a non-reduced fraction

4 Consider, for example, $3 + (-2)$ and $3 - (-2)$

5 Use definition of rational numbers

6 Subdivide the n-gon as for Exercise 1E question 6

7 $a = 1$, $b = -1$, $c = 1$

8 Factorise and use an exhaustive proof

9 Construct further lines to find isosceles triangles; construct a proof using knowledge of angles in triangles

10 Consider n^3 where $n = 3k$ or $n = 3k \pm 1$

11 **a** This does not work in the forward direction, e.g. $a = 4$, $b = 0.5$

b This does not work in either direction, e.g. $a = \sqrt{2}$, $b = \sqrt{2}$ so $ab = 2$ **or** $ab = \pi$, $a = \pi$, $b = 1$

12 Use $a = n - 1$, $b = n$ and $c = n + 1$

13 Use $a = 2m + 1$ and $b = 2n + 1$

14 **a** e.g. $n = 1601$ **b** factorise n^{2-1}

15 Factorise and use an exhaustive proof considering whether $a + b$ is odd or even