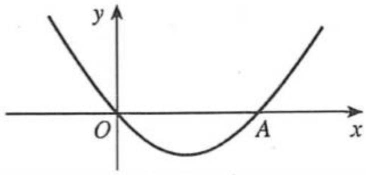
Transforming graphs (answers at the end)

Do not use a calculator in this exercise.

1 Starting with the graph of $y = \frac{1}{x}$, use a translation to sketch the graph of $y = \frac{1}{1+x}$. Hence sketch the graph of $y = \frac{1}{1+\frac{1}{3}x}$.

2 The diagram shows the graph of $y = f(x)$. The curve passes through the origin O and the point A with coordinates $(a, 0)$, where a is a positive constant. Sketch, on separate diagrams, the graphs of



(a) $y = f(x+a)$ (b) $y = f(-x)$ (OCR)

3 The graph of $y = \frac{1}{x}$ is first stretched by a factor of 3 in the x-direction and then stretched by a factor of $\frac{1}{3}$ in the y-direction. What is the effect on the original curve?

4 The straight line $y = ax + b$ is transformed by two translations. One translation is by 4 units in the positive x-direction and the other is by 7 units in the positive y-direction. Given that the equation of the transformed line is $y = 6x - 27$, find the values of a and b.

5 The graph of the straight line $y = x$ is transformed as follows:

translation by 4 units in the positive x-direction,

followed by stretch in the y-direction with scale factor 2,

followed by reflection in the x-axis.

Find the equations of the graphs which result after each transformation.

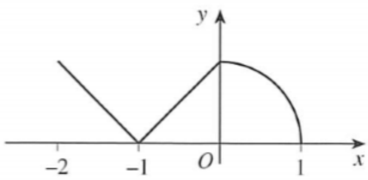
6 If the graph of $y = f(x)$ is reflected in the x-axis and then in the y-axis, what is its new equation? If the graph is the same as the original, what type of function is $f(x)$?

7 The diagram shows the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 1$. Outside this interval $f(x)$ is zero.

Sketch, on separate diagrams, the graphs of

(a) $y = f(x+1)$,

(b) $y = -f(3x)$. (OCR)



8 Describe clearly the transformations which convert the graph of $y = f(x)$ into

(a) $y = af(x)$, (b) $y = f(x) + a$,

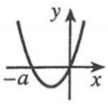
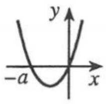
(c) $y = f(x+a)$, (d) $y = f(ax)$,

where a is a positive constant.

9* Draw a sketch of the graph of an even function $f(x)$ which has a derivative at every point. Let P be the point on the graph for which $x = p$ (where $p > 0$) and draw the tangent at P on your sketch. Also draw the tangent at the point P' for which $x = -p$.

(a) What is the relationship between the gradient at P' and the gradient at P? What can you deduce about the relationship between $f'(p)$ and $f'(-p)$? What does this tell you about the derivative of an even function?

(b) Show that the derivative of an odd function is even.

2 (a)  (b) 

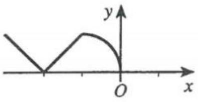
3 The curve remains the same.

4 $a = 6,\ b = -10$

5 $y = x - 4$, $y = 2x - 8$, $y = -2x + 8$

6 $y = -f(-x)$; $f(x)$ is an odd function.

7 (a)



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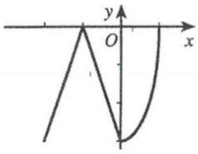

8 (a) Stretch, factor a in the y-direction

(b) Translation a units in the y-direction

(c) Translation $-a$ units in the x-direction

(d) Stretch, factor $1/a$ in the x-direction

9 (a) The gradient at P' is the negative of the gradient at P. So $f'(-p) = -f'(p)$. The derivative of an even function is odd.